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# DIRECT DETERMINATION OF THE SOLAR PHYSICAL COORDINATES FROM PHOTOHELIOGRAMS

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**Abstract.** The sunspots observation by photoheliograms in time for two weaks, can be used for direct determination of solar physical coordinates $B_0, P$ if a level is used for determination of horizontal line on the plate. The method is presented, with an example.

## 1. INTRODUCTION

The application of level in photo-heliograph, for the determination of angle $\chi$ between the daily parallel and the basis of the photo-frame, gave better accuracy in determination of spot coordinates (Tomić, 1979, 2005). It is not the complete gain however, if only the level is used. Two additional possibilities appear:

(1) Determination of sunspots coordinates from photos on which solar disc is not totaly visible.
(2) Direct estimation of physical coordinates of the Sun.

Here we present the both advantages of method which use the level.

## 2. DETERMINATION OF COORDINATES ON PARTIALY VISIBLE DISC

The procedure of observation, measure on shoot and calculation of angle $\chi$ are given in references Tomić, 1976 and Tomić 1979. Basic concept of the method is contained in Figs. 1 and 2. The idea for application to partially visible solar disc with analysis of accuracy, previously were presented and published as abstract (Tomić, 1996).

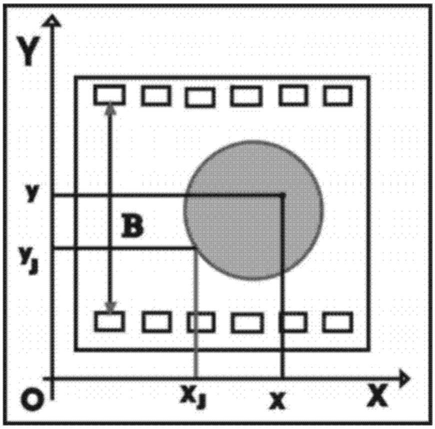


**Figure 1:** Measured quantities in the arbitrary chosen frame.

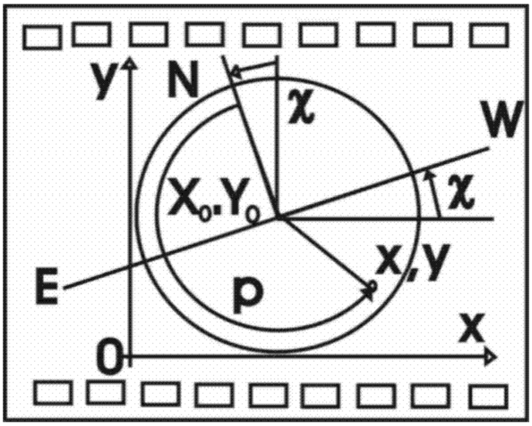


**Figure 2:** Angle $\chi$ between daily parallel and horizon, which needs to be calculated.

Here we present sunspots coordinates *B, L*, their maximal errors according to model *dB, dL*, averaged difference for obtained coordinates for shoots obtained in regime A – whole disc visible, and regime B – partially visible limb, into a series with *n* shoots, $\Delta\overline{B}$ , $\Delta\overline{L}$ . The focal length of photoheliograph is given too in Table 1. In Fig. 3 is presented a shoot with partially visible solar limb.

In Table 1 is shown that method which used a level, gives possibility for determination of coordinates from shoots with partially visible solar limb with same accuracy. (Limb is needed for the determination of the radius. Procedure gives the same accuracy on both images, by using 20 to 50 points on the limb, and image magnification of 18 to 20 times.) Coordinates in series differs mutually less than calculated theoretical error value. Errors are depending from the central angle.

Series follows spot from the eastern to the western solar limb, with the solar physical coordinates $B_0 \approx +0.5^0$ and $P_0 \in (-13^0;-9^0)$, according to ephemeris. In the Greenwich listing the spot number were 833, and according to the Solnechnye dannye 57.

The comparison of Greenwich measures and Solnechnye dannye with our data for the same spot is presented graphically in Fig. 4. Mean latitudes are: –21.2 (GR), –21.1 (SD), –21.5 (TA) in degrees. In all three cases obtained value lies within the error bars for the used method of determination. In other words, mean latitude was good measured. The variations were the smallest (approximately twice) in our measures.

For longitude were obtained: +3.6 (GR), +4.3 (SD), +5.9 (TA) in degrees. Here variation was the smallest amplitude in our measures, too.

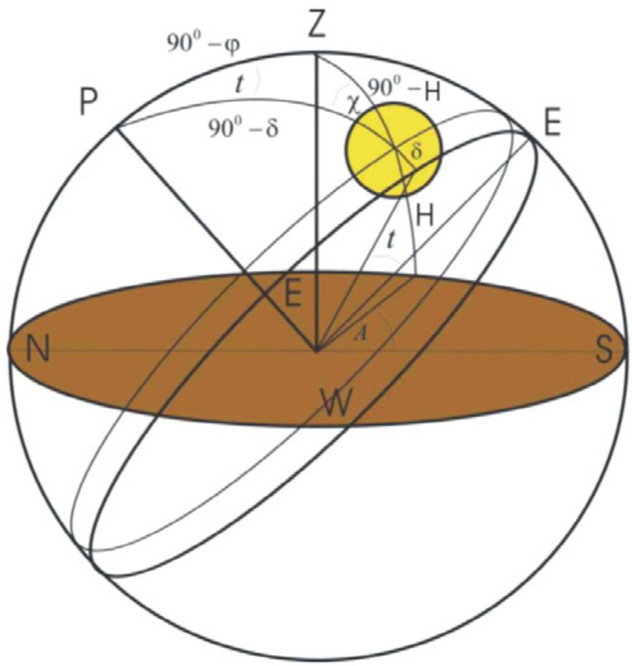


**Figure 3:** Quantities which determine angle $\chi$ .

In the longitude were obtained: +3.6 (GR), +4.3 (SD), +5.9 (TA) in degrees. Here the variation was with the smallest amplitude in our measures. Greenwich data were with the biggest variation.

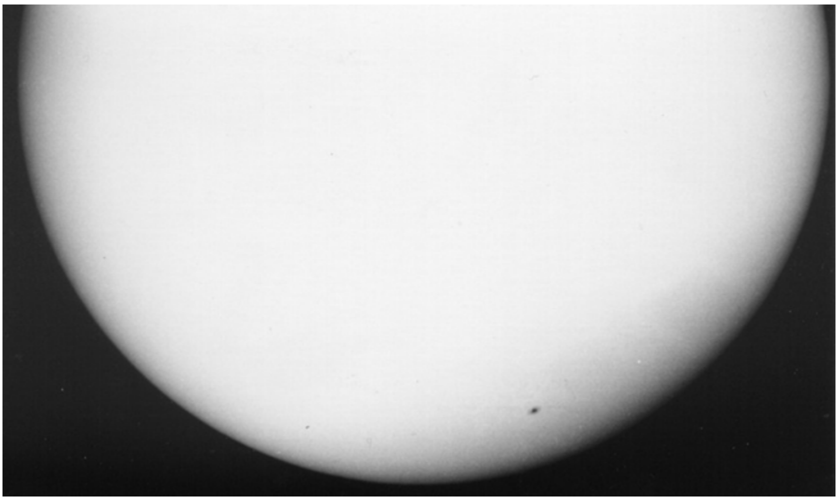

**Figure 4:** Photo with partially visible solar limb.

## 3. DIRECT DETERMINATION OF THE SOLAR PHYSICAL COORDINATES

The normal line to daily parallel through the center of solar disc presents a projection of the celestial meridian through the center of the Sun. After calculation of angle $\chi$, coordinates of the center and radius of solar disc $X_0, Y_0, R$, spots coordinate $x, y$ can be calculated in the coordinate frame with origin in the solar disc center with axis $y_1$ along celestial meridian, as follows:

$$x_1 = \frac{x - X_0}{R}\cos\chi + \frac{y - Y_0}{R}\sin\chi \tag{1}$$

$$y_1 = -\frac{x - X_0}{R}\sin\chi + \frac{y - Y_0}{R}\cos\chi \ . \tag{2}$$

Transformation to the frame with origin in solar disc center, with the axis y parallel to the solar rotation axis, realizes by rotation for angle $P$ of inclination of the solar axis of rotation towards celestial meridian:

$$x_2 = x_1 \cos P + y_1 \sin P \tag{3}$$

$$y_2 = -x_1 \sin P + y_1 \cos P \, . \tag{4}$$

In this transformation was used the angle $P$ which value is not known. For this reason we use a proposal that in this time interval the solar axis of rotation is fixed according to normal line toward ecliptic. By angular inclination of solar axis to the celestial meridian, the solar center is projected along this axis. Now we use the fact that central distance does not change by the rotation of coordinate frame:

$$x^2 + y^2 = x_1^2 + y_1^2 = x_2^2 + y_2^2 = r^2 \tag{5}$$

Further we apply this procedure to the spots photographed two weaks during their transit along the solar disc. For the same spot (photographed in different datum) we calculate the central distance $r$. Now we choose positions which are near to E limb (order number $k$) and near to W limb (order number $n$), at approximately equal central distances and with the smallest difference $\Delta r$. For the pair with the smallest mutual difference $\Delta r$ then we calculate the inclination of the line trough these two positions to the $x$ –axis, as:

$$\tan P = [(y_1)_n - (y_1)_k] / [(x_1)_n - (x_1)_k] \tag{6}$$

From this value (as the first approximation) are calculated sin $P$ and cos $P$ and inserted into Eqs. (3-4) to calculate $x_2$, $y_2$.

From these can be developed values $B, B_0$, because orthogonal and heliographic coordinates are connected by relations:

$$x_3 = \cos(B + B_0) \cdot \sin L \tag{7}$$

$$y_3 = \sin(B + B_0) - \sin B_0 \cdot |\sin L| \ . \tag{8}$$

Adapted for application of the least squares method obtains:

$$y_3 = b \cdot |x_3| + a \tag{9}$$

$$a = \sin(B + B_0), \quad b = -\sin B_0 / \sqrt{1 - a^2} \ . \tag{10}$$

Solving for *a, b* separate for series with positive and series with negative values of x can be determined $b_+, b_-$ and calculated the correction of inclination for the axis of rotation:

$$\Delta P = -[\arctan(b_+) - \arctan(b_-)]/2 \ . \tag{11}$$

This procedure we applied to the previously presented spot No 833 in Greenwich listing for solar cycle No 20.

Here are *x, y* (in coordinate frame with y - axis orthogonal to the horizon and arbitrary chosen position of coordinate origin) and $X_0$, $Y_0$, $R$ given in millimeters, angle $\chi$ in degrees. Values $x_1$, $y_1$ are given as a fraction of radius R.

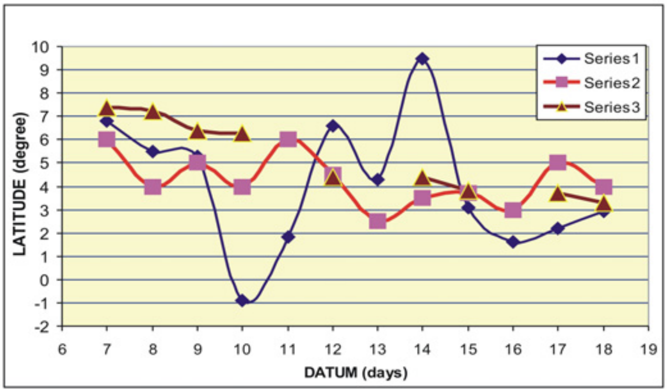


**Figure 5:** Latitude data for Greenwich measures (1-GR), Solnechnie daniya (2-SD) and our (3-TA).

The comparison of obtained coordinates for whole disc (regime A) and for partially visible solar limb (regime B) is presented in Table 1. By theoretical model were calculated errors *dB, dL* and from measures mean differences in series of *n* shoots $\Delta\overline{B}$, $\Delta\overline{L}$.

**Table 1.** Spot coordinates B, L, calculated errors dB, dL and mean differences in series $\Delta\overline{B}$ , $\Delta\overline{L}$ for whole disc (A) and for partially visible solar limb (B)

| Order number | Series | Datum | B $(^0)$ | dB $(^0)$ | $\Delta\overline{B}$ $(^0)$ | $\Delta\overline{L}$ $(^0)$ | dL $(^0)$ | L $(^0)$ | $\ell$ $(^0)$ | n | Regime | **F** (mm) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 42 | 7.4992 | -21.24 | 0.97 | 0.14 | 0.78 | 2.98 | 7.41 | -75.3 | 5 | A | 2000 |
| 2 | 43 | 8.2925 | -21.78 | 0.90 | 0.32 | 0.32 | 1.60 | 6.72 | -62.3 | 4 | A | 2000 |
| 3 | | 8.3464 | -21.24 | 0.90 | 0.41 | 0.15 | 1.53 | 6.75 | -62.2 | 5 | B | 3200 |
| 4 | 44 | 8.5476 | -21.57 | 0.86 | 0.32 | 0.76 | 1.34 | 7.15 | -59.2 | 4 | A | 2000 |
| 5 | | 8.5498 | -21.68 | 0.87 | 0.16 | 0.54 | 1.39 | 6.39 | -59.9 | 6 | B | 3300 |
| 6 | 45 | 9.5381 | -21.68 | 0.73 | 0.11 | 0.14 | 0.90 | 6.47 | -46.8 | 5 | A | 2000 |
| 7 | | 9.5411 | -21.64 | 0.73 | 0.15 | 0.05 | 0.90 | 6.36 | -46.9 | 3 | B | 3300 |
| 8 | 47 | 10.4577 | -22.39 | 0.57 | 0.32 | 0.24 | 0.68 | 6.33 | -34.7 | 5 | B | 3500 |
| 9 | 48 | 12.6071 | -21.42 | 0.15 | 0.18 | 0.19 | 0.42 | 4.57 | -8.1 | 6 | A | 2000 |
| 10 | | 12.6097 | -21.41 | 0.22 | 0.10 | 0.16 | 0.55 | 4.42 | -8.2 | 6 | B | 3300 |
| 11 | 49 | 14.3248 | -21.40 | 0.23 | 0.11 | 0.08 | 0.73 | 3.78 | 14.0 | 4 | A | 2000 |
| 12 | | 14.3274 | -21.53 | 0.22 | 0.04 | 0.19 | 0.43 | 3.79 | 14.0 | 3 | B | 3000 |
| 13 | 50 | 14.4585 | -21.45 | 0.32 | 0.01 | 0.4 | 0.48 | 4.34 | 16.3 | 5 | A | 2000 |
| 14 | | 14.4604 | -21.42 | 0.33 | 0.09 | 0.19 | 0.49 | 4.56 | 16.7 | 7 | B | 3300 |
| 15 | 51 | 15.4295 | -21.40 | 0.48 | 0.33 | 0.25 | 0.56 | 4.09 | 28.8 | 4 | A | 2000 |
| 16 | | 15.5311 | -21.29 | 0.48 | 0.25 | 0.10 | 0.56 | 3.92 | 28.7 | 3 | B | 3300 |
| 17 | | 15.4469 | -20.84 | 0.49 | 0.11 | 0.24 | 0.56 | 4.09 | 29.7 | 2 | A | 2000 |
| 18 | | 15.4585 | -20.65 | 0.52 | 0.48 | 0.06 | 0.56 | 4.55 | 29.7 | 3 | B | 3200 |
| 19 | | 15.5314 | -21.86 | 0.52 | 0.15 | 0.11 | 0.62 | 3.91 | 30.0 | 7 | A | 2000 |
| 20 | 52 | 17.5093 | -20.91 | 0.82 | 0.16 | 0.61 | 1.15 | 3.11 | 55.3 | 3 | A | 2000 |
| 21 | | 17.5138 | -21.28 | 0.83 | 0.19 | 0.07 | 1.20 | 3.74 | 56.0 | 4 | B | 3300 |
| 22 | 53 | 18.6085 | -21.84 | 0.94 | 0.15 | 0.35 | 2.24 | 3.28 | 70.0 | 4 | A | 2000 |

Measured data (Datum, x, y), from these calculated position of solar centre and radius ($X_0$, $Y_0$, R), calculated angle $\chi$ and spots heliocentric coordinates within the frame with y-axis parallel to the celestial meridian ($\chi$, $x_1$,$y_1$), for spot observed within the period June 7–18, 1977, are presented in Table 2. It is obvious that as pairs are convenient for the use the positions No. 2 and 9, what results in $P = -10.7^0$. With the Eqs. (3-4) one obtains values presented in Table 3. Values from columns 3 and 4 are presented in Fig. 6a.

**Table 2.** Measured data (Datum, x, y), calculated data ($X_0$, $Y_0$, R, $\chi$) and spots heliocentric coordinates ($x_1$,$y_1$) within the frame with y – axis parallel to the celestial meridian

| | Datum | $x$ | $y$ | $X_0$ | $Y_0$ | R | $\chi$ | $x_1$ | $y_1$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 7.3122 | 128 | 83 | 219.61 | 224.09 | 172.76 | 48.179 | -0.9622 | -0.1494 |
| 2 | 8.5478 | 128 | 310.4 | 230.52 | 238.9 | 172.76 | -45.227 | -0.8880 | -0.1785 |
| 3 | 9.5407 | 97.6 | 303 | 208.72 | 246.65 | 172.76 | -43.087 | -0.7429 | -0.2156 |
| 4 | 10.4574 | 101 | 219 | 270.63 | 263.28 | 172.76 | -9.876 | -0.5965 | -0.2720 |
| 5 | 12.6094 | 101 | 305.2 | 205.85 | 322.668 | 172.76 | -50.365 | -0.3034 | -0.3470 |
| 6 | 14.4608 | 128 | 226.7 | 141.28 | 344.728 | 172.76 | -11.909 | 0.1869 | -0.4265 |
| 7 | 15.4566 | 97.6 | 112.6 | 225.9 | 202.75 | 172.76 | -10.100 | 0.4050 | -0.4374 |
| 8 | 17.5141 | 101 | 128.8 | 279.097 | 341.46 | 172.76 | -37.983 | 0.7097 | -0.4851 |
| 9 | 18.6087 | 101 | 130.7 | 238.038 | 280.473 | 172.76 | -40.93 | 0.8132 | -0.5000 |

**Table 3:** Spot coordinates within the frame with the preliminary determined axis of rotation, and constants a,b determined by the least squares method

| Datum | $r$ | $x_2$ | $y_2$ | | |
|---|---|---|---|---|---|
| 7.31223 | 0.9737 | -0.9175 | -0.3255 | D 4.9737 | |
| 8.54777 | 0.9058 | -0.8394 | -0.3403 | | |
| 9.54068 | 0.7736 | -0.6900 | -0.3498 | | |
| 10.45735 | 0.6556 | -0.5356 | -0.3780 | $D_1$ 0.421966 | b 0.085825 |
| 12.60938 | 0.4609 | -0.2337 | -0.3973 | | |
| 14.4608 | 0.4657 | 0.2628 | -0.3844 | | |
| 15.4566 | 0.7207 | 0.4792 | -0.3546 | $D_2$ -2.02059 | a -0.41098 |
| 17.51405 | 0.8596 | 0.7855 | -0.3449 | | |
| 18.60868 | 0.9113 | 0.8919 | -0.3403 | | |

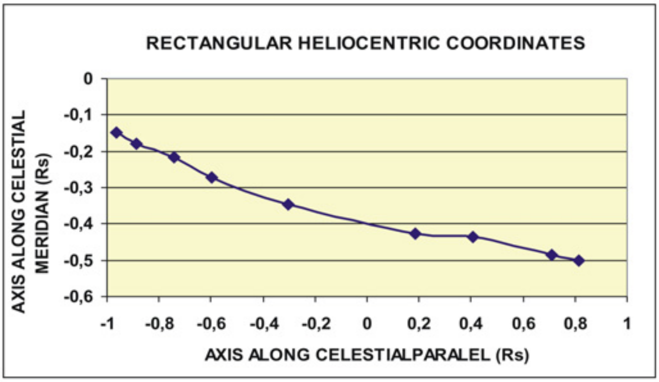


**Figure 6a:** The path of a spot within the frame with y axis normal to the daily parallel.

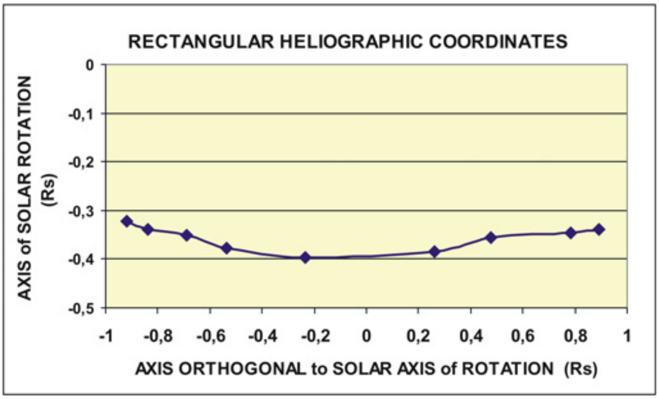


**Figure 6b:** The path of a spot within the frame with the axis y parallel to solar axis of rotation, determined in this procedure.

For the first five and the rest four positions, used separately, angles differ 2.54 degrees and the correction is equal to 1.27 degrees. Added to the calculated values it gives:

June 10.0: $P = -10.7^0 - 1.27^0 = -11.97^0$, value given in ephemeris: $P = -12.0^0$
June 16.5: $P = -10.7^0 + 1.27^0 = -9.43^0$, value given in ephemeris: $P = -9.3^0$.

The agreement is very good, what suggests such application. The essential benefit if the level is used present accuracy in determination of spot coordinates. Moreover, the same data can be used for additional purposes, as presented here.

## 4. CONCLUSIONS

This procedure can be interesting as a way of temporary indirect control of the angular positions for movable parts of camera, too. Deviation bigger than 0.1 degree (accuracy limit in our measure) appears in estimation as doubled systematic difference between eastern and western spot position by 100/2000 mm aperture of camera. For a better estimation of the spot positions, in order to follow meridian motion which amplitude is very small, it can be very useful.

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