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STABILITY AND EVOLUTION OF MAGNETIC ACCRETION DISK

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Abstract: In this paper we consider the magneto-hydrodynamic of the hot advection accretion disk. We investigate the interaction between stream and magnetic field. Appearance and activity of the instabilities in the stream is discussed. Here we will show our results for 2D radial structure of disk and local warm in disk. How the flow is develop in (r, φ)-plane on disk. We show the form of conditions for destroying of us disk to the inner edge.

1. INTRODUCTION

Magneto-rotational instability (MRI) is important mechanism for keeping the equilibrium of hot, advective magnetizing disk. Hydrodynamic turbulence (HDT) gives general contribution for release of energy in disk, but MRI has basic contribution, too. In the proposed model both this mechanisms are with great efficacity, but viscous dissipation dominated and this helping on disk to crossing all magnetosphere. Presented is global solution of the model, and also local in thin ring with Alphen velocity v_a and sonic velocity v_s closely to constants.

This is approximate solution, but provides a good idea for stability and evolution of disk.

2. MHD MODEL: GLOBAL AND LOCAL

Constructed is non-stationary, non-axisymmetric, one-temperature MHD model of Keplerian accretion disk with advection in the normal dipole magnetic field of the central object.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{1}$$

$$\nabla \cdot B = 0 \qquad\qquad \nabla \cdot v = 0 \tag{2}$$

$$\frac{\partial v}{\partial t}+v\cdot\nabla v=-\frac{1}{\rho}\nabla p-\nabla\Phi+(\frac{B}{4\pi\rho}\cdot\nabla)B+\vartheta\nabla^2 v \quad , \qquad \Phi=\frac{GM}{r-r_g} \tag{3}$$

$$\frac{\partial B}{\partial t}=\nabla\times(v\times B)+\eta\nabla^2 B \quad , \qquad \eta=\frac{\eta_m}{\rho}=\frac{c^2}{4\pi\sigma} \qquad v_a^2=\frac{B^2}{4\pi\rho} \tag{4}$$

$$\rho T\frac{\partial S}{\partial t}-\frac{\dot{M}}{2\pi r}T\frac{\partial S}{\partial r}=Q^+-Q^-+Q_{mag}\,,\ Q_{mag}=\frac{\eta}{4\pi}(\nabla\times B)^2_\varphi \quad r_g=\frac{2GM}{c^2} \tag{5}$$

$$p=p_r+p_g+p_m\,, \qquad Q_{adv}=-\frac{\dot{M}}{2\pi r}T\frac{\partial S}{\partial r}=-2H\rho v(-\frac{3}{2}\frac{B^2}{8\pi\rho r}) \tag{6}$$

$$Q^+=\frac{W_{r\varphi}}{2}(r\frac{\partial\Omega}{\partial r})=H\rho\vartheta(r\frac{\partial\Omega}{\partial r})^2 \qquad Q^-=\frac{c}{3\chi\rho}\nabla(aT^4)$$

Here v is velocity of flux; ρ - mass density; B - magnetic field; Φ - gravitational potential; p-pressure; $\vartheta=\alpha v_s H$ -kinematical viscosity; $\eta=\alpha_m v_s H$ - magnetic viscosity; Q^+ - viscosity dissipation; Q_{adv} - advection dissipation or cooling; Q^- - radiative cooling.

We accept that:

$B_z=\frac{\mu}{r^3}$ for equatorial plane and is independence from φ and t

$\vec{v}=(v_r,\Omega_k=\sqrt{\frac{GM}{r(r-r_g)^2}},0)$ because disk is Keplerian

$\eta=v_s H\,;\alpha_m=1\,;T=T_{vir}=\frac{GMm_i}{kr}$ because disk is advective.

We used cylindrical co-ordinates (*r, φ, z*; *t*). Used are the mass continuity, magnetic flux conservation, equation of motion, angular momentum conservation, hydrostatic balance, the three components of magnetic induction, heat balance and assumed approximations. Here we will use obtained analytical solution. The results are obtained only along *r*, because we assume periodical dependencies for variables *φ* and *t* with coefficients dependent on *r*.

2.1. Global Solution

Global solution is obtained earlier in (Iankova and Filipov, 2004) in dimensionless variables:

$$f_1(x) = \frac{4+c_2}{6x^6} + \frac{1+c_1+c_3}{x^{15/2}}\left[1-(x-x_g)\right] + \frac{2-c_2}{6}$$

$$f_2(x) = (c_2+4)(x-1)+1$$

$$f_3(x) = \frac{c_6}{7}x^7 + \frac{x^3}{3}\left(c_7 - \frac{(3+c_2)}{2}c_5\right) - \frac{(3+c_2)}{6x^3} -$$
$$-\frac{1}{2}c_5(1+c_1+c_3)x^{3/2}(x-x_g-1)+$$
$$+\frac{1+c_1+c_3}{2x^{9/2}}(x-x_g-1)+1-\frac{c_6}{7}-\frac{c_7}{3}+\frac{(3+c_2)(1+c_5)}{6}$$

$$f_4(x) = (1-c_{13}-c_{14})x^2 + \frac{c_{13}}{x^6} + c_{14}(x-x_g)\,; \qquad f_5(x) = \frac{1+c_4}{x} - c_4$$

$$f_6(x) = -\frac{c_{10}x^2}{2c_9} - (x-1) - \frac{c_8}{2c_9x^{7/2}} - \left(\frac{3c_8}{2c_9} - \frac{1}{c_9}\right)\frac{1}{x^{9/2}} + \frac{1+c_1}{2c_9x^5} + \frac{2\alpha c_{11}}{c_9x^{11/2}} +$$
$$+\frac{9\alpha c_{11}}{4c_9x^{13/2}} + \frac{\alpha c_{12}}{c_9x^{17/2}} + \left[\left(\frac{3c_8}{2c_9} - \frac{1}{c_9}\right)\frac{1}{x^{9/2}} - \frac{9\alpha c_{11}}{4c_9x^{13/2}} - \frac{\alpha c_{12}}{c_9x^{17/2}}\right](x-x_g)+$$
$$+\left[\frac{c_8}{2c_9x^{7/2}} - \frac{2\alpha c_{11}}{3c_9x^{9/2}} - \frac{1+c_1}{2c_9x^5} - \frac{2\alpha c_{11}}{c_9x^{11/2}}\right]\frac{1}{(x-x_g)} + \frac{2\alpha c_{11}}{3c_9x^{9/2}}\frac{1}{(x-x_g)^2} +$$
$$+\frac{2c_9-c_{10}}{2c_9}$$

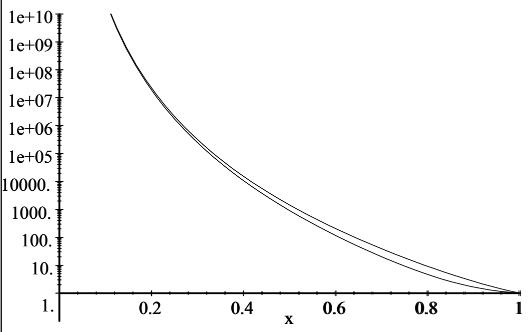
$$f_7(x) = -\frac{c_2+4}{c_1 x} + \frac{c_{16}+c_{12}}{c_1 x^5} + \left[-\left(\frac{c_3}{2c_4} + c_3 \right) \frac{1}{c_1 x^{3/2}} + \frac{c_{17}}{c_1 x^{9/2}} \right] \frac{1}{x - x_g} -$$

$$- \frac{c_3}{c_4 c_1 x^{1/2}} \frac{1}{(x-x_g)^2} + 1 - \frac{c_{17}}{c_1} + \frac{c_3}{c_1} + \frac{c_2+4}{c_1} - \frac{c_{16}+c_{12}}{c_1} + \frac{3c_3}{2c_4 c_1}$$

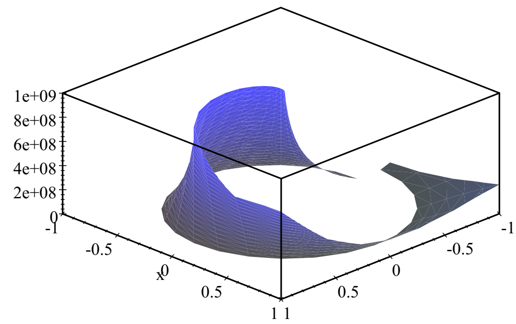
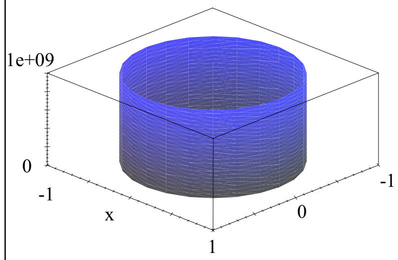
$$f_8(x) = \frac{c_{15} x^2}{2c_4 c_{11}} + \frac{c_4+1}{c_4 c_{11}} x - \frac{1}{c_4 x} - \frac{c_{16}+c_{12}}{3c_4 c_{11} x^3} - \frac{1+c_1+c_3}{c_4 c_{11} x^{1/2}} (x - x_g - 1) +$$

$$+ 1 - \frac{c_{15}}{2c_4 c_{11}} - \frac{c_4+1}{c_4 c_{11}} + \frac{1}{c_4} + \frac{c_{16}+c_{12}}{3c_4 c_{11}}$$

Here functions f_i are dimensionless parameters. Constants c_i is the corresponding combination of ρ_0, v_{r0}... That is parameters on outer edge of the disk. They depend from inflow. This give the radial structure for the point of inflow in the moments $t \approx 0$ and t=1P~Ω_0^{-1} after the spreading of the disc.



This is the distribution of dimensionless density $f_1(x)$ for $\varphi=0$ **in the two moments.**
We can see on figure how the intensity of the inflow increases for one period.

	
This is the distribution of dimensionless density $f_1(X, Y)$ in non-axial-symmetric MHD model. We can see in Fig. the beginning of the spiral about the outer edge as minimum of the function at the density of 0.9*x*.	**This is the distribution of dimensionless density $f_1(X, Y)$ in plane of the disk for moment t ≈ *0*.** The matter is accumulating mainly in the ring on orbit ~0.8*x*, but the spreading is in progress.

This solution we will use to obtain the local warm up of the disk, which is basis for appearing and feeding of instability.

2.2. *K* Warm Up In Disk

Local solution is obtained earlier in (Iankova and Filipov, 2003)

$$K = \frac{\text{warming}}{\text{cooling}} = \frac{\vartheta(r\dfrac{\partial\Omega}{\partial r})^2 + \dfrac{\eta}{4\pi\rho}\left(\dfrac{B_r}{H} + \dfrac{3\mu}{r^4}\right)^2}{\dfrac{c}{\chi\rho H^2}(v_s^2 - RT - \dfrac{v_a^2}{2\alpha_m^2}) + \dfrac{3}{2}v_r\dfrac{v_a^2}{r}}$$

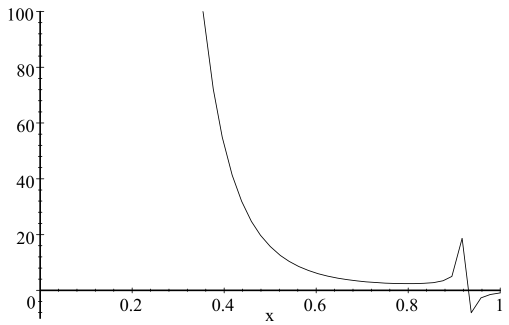
$$i\omega = \left(\Omega - \frac{v_r}{r}K\right) + \frac{1}{v_r}\left(\frac{v_a^2}{2} - v_s^2\right)\frac{1}{r}\left(1 \pm \sqrt{1 - \frac{4\Omega r^2}{v_s H} + \frac{4v_r r}{v_s H}K}\right) - \frac{1}{v_r}\Omega^2 r$$

$$ik_\varphi = K - k_r r = K - 1 \mp \sqrt{1 - \frac{4\Omega r^2}{v_s H} + \frac{4v_r r}{v_s H}K}$$

$$k_r = \frac{1}{r}\left(1 \pm \sqrt{1 - \frac{4\Omega r^2}{v_s H} + \frac{4v_r r}{v_s H}K}\right).$$

The global model gives us distributions of the characteristics of the flow on the disc, but instabilities are supported from the local warm *K*. When we obtain him from the characteristics, it is necessary to include the global distributions. So we get the distribution of local warming in all the disk.

$$K = \frac{\alpha a_2 \left[\frac{x}{4\left(x - x_g\right)^2} + \frac{x^2}{\left(x - x_g\right)^3} + \frac{x^3}{\left(x - x_g\right)^4} \right] + \alpha_m \left(a_6 x^6 + a_7 x^5 + a_8 x^4 \right)}{-2 \frac{a_4}{\alpha_m{}^2} x^{15} - \frac{a_4}{\alpha_m{}^2} x^{13} + a_3 x^9 - a_5 x^8 + 2 a_1 x^6 + a_1 x^4}$$



Distribution of the local warm in disk *K*(*x*).
We observe interesting element on the outer edge in the beginning of the distribution, which probably corresponds to the rupture in the spiral.
We can see the part of disk in thermal balance and the part without effective cooling.

2.3. Condition (Q_{mag}/Q^{+}) < 1

$Q_{\mathrm{mag}}/Q^{+} = 1$ locate the radius of destruction of the disc in magnetosphere (Campbell, 1998; Campbell and Heptinstall, 1998).

$$\left(\frac{B_r}{H} + \frac{3\mu}{r^4}\right)^2 \leq 4\pi\alpha\rho H \left(r \frac{\partial\Omega}{\partial r}\right)^2$$

$$\frac{1}{4\pi\rho}\left(\frac{B_r}{H} + \frac{B_z}{r}\right)^2 \approx \frac{\langle v_a \rangle^2}{r^2} \leq \alpha H \left(-\frac{3}{2}\Omega_k\right)^2 \leq \frac{9}{4}\Omega_k^2$$

$$\langle v_a(r) \rangle^2 \leq \frac{9}{4} \langle v_\varphi(r) \rangle^2$$

The local condition for the radius of destruction obtained in such form is convenient to use for the evaluation of R_d.

3. CONCLUSION

The results give us the possibility to obtain a general summarized picture for physical processes in disk: We can see the beginning of the spiral. The warm at inner layer, where instabilities are supported from him. We see the distributions in the moment $t \approx 0$ and we can compare the behavior of the flow in two moments.

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