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REMARKS ON THE MASS DISTRIBUTION IN STELLAR SYSTEMS

SLOBODAN NINKOVIĆ[1]

[1]*Astronomical Observatory, Volgina 7, 11160 Belgrade-74, Yugoslavia*
E–mail sninkovic@aob.bg.ac.yu

Abstract. A general consideration of mass distribution in stellar systems is presented.

1. General

It is rather well known that in the modern approach, the starting point in the study of the mass distribution within a stellar system is the observed profile. The alternative consisting of ascribing a given theoretically based profile to the stellar system under study belongs to the past. As an example one can mention Plummer's (e. g. 1915) papers where a particular theoretically based density law (Schuster's polytrope) was attributed to the observed globular clusters. However, a number of papers originating mainly from the middle of the twentieth century clearly indicated the importance of introducing empirical density laws (e. g. de Vaucouleurs, 1948; King, 1962). The modern equipment, doubtlessly, enables a sophisticated treatment of observational data, in particular to obtain a satisfactory trend between the observational points. In this way one has nowadays many empirical profiles (surface density is ment) which rather well reflect the real mass distribution within a class of stellar systems, i. e. concrete stellar systems (e. g. Dehnen, 1993 and references therein). However, one must take into account that the region of a stellar system available to the observations is limited, in other words there are always points evading a direct evidence. In such a situation the only way of obtaining the surface density is to calculate it by using the fitting formulae (model). Then one can have unusual outcomes: a model used for the purpose of fitting the observational results can result in an infinite space density at the system centre, or likewise this can be the case with both the surface density and the space one. For instance, Dehnen's model (mentioned above), except one special case, yields infinite values at the centre for both densities. On the other hand de Vaucouleurs' model (also mentioned above), though yielding a finite central surface density, results in an infinite central space density! No matter what kind of physical meaning is ascribed to such results, it is clear that they cannot be realistic. For example, one may consider the possibility of a massive central black hole. Then the main result would be its mass. However, as very well known, the mass value for a black hole clearly determines its radius and, as a consequence, it follows that the black-hole density is finite! There are also other disadvantages of models yielding central singularities, for instance, though the total mass is finite, the total potential energy can be infinite (e. g. Tremaine et al., 1994).

Problems can also arise when the outer boundary is studied. Usually for the purpose of avoiding the well-known difficulties it is assumed that the stellar system under study is isolated, i. e. that the density in the surrounding space is zero. In order to make this presentation clear enough the present author will take into consideration the example of a spherically symmetric stellar system. In such a situation there is the very clear notion of the maximal apocentric distance. Obviously, its amount must be finite, but it need not be specified. The latter case corresponds to a model with an infinite limiting radius of the (stellar) system. In the reverse case, i. e. if the limiting radius is finite, it must be equal to the maximal apocentric distance. Then, as a consequence, the density on the inner side of the outer sphere (that of the radius equal to the limiting one) must be non-zero, i. e. there will be a discontinuity. Some authors (e. g. Casertano, 1983; even the present author was once among them - e. g. Ninković, 1992) reject such a possibility. However, there is no reason to be afraid of this discontinuity, it is practically the consequence of the artificial assumption that the surrounding space is empty.

Therefore, the present author's position is that the outer boundary of a stellar system in a model of it can be both finite and infinite, but that in the former case the discontinuity at the outer boundary is unavoidable, whereas in the latter one a model is meaningful only if it yields finite values for both the total mass and the total potential energy. Thus in this, latter, case the boundary discontinuity vanishes since the system density reaches zero in the infinity. Perhaps it may seem that the same line of reasoning is valid for the central density, i. e. that an infinite central density merely means that the central-density value in a given model is not specified (in other words it can always exceed a given limit), but in addition to the difficulties indicated above, there are also ones concerning the computer programming in the presence of a central singularity (say the orbits), not to speak about the velocity distribution, for instance how to understand an isotropic velocity distribution extending to the centre when due to the central density singularity the orbits of individual stars cannot include the centre itself. The existence of a central density singularity can be justified to some extent for the case of a central black hole since then the gravitational potential in the immediate surroundings of the centre is that of point mass. However, as already remarked, the black-hole mass determines its radius, in other words since the mass must be finite, as a consequence the black-hole density (central density) will be finite. In addition, clearly, for the case of point-mass potential highly eccentric orbits of zero angular momentum are excluded. Consequently, in the immediate surroundings of the centre the velocity distribution can be hardly isotropic.

2. Some concrete proposals

In an earlier paper (Ninković, 1998) the present author considered a special case of the generalised Schuster density law. In particular, for the space density the following formula is borne in mind

$$\rho(r) = \frac{\rho(0)}{[1 + (r/r_c)^2]^{i/2}} , \tag{1}$$

where r is the distance to the centre of the stellar system under study (assumed to be spherically symmetric), ρ is its space density, r_c is a constant parameter (system scale length or core radius), i being a non-negative integer. As easily seen, the case $i = 5$ corresponds to the classical Schuster density law. However, from the empirical point of view the special cases $i = 2$, $i = 3$ and $i = 4$ are of more interest. As indicated in the mentioned paper (Ninković, 1998), the first of them corresponds to the quasi-isothermal-sphere case applied most frequently for the purpose of describing the mass distribution anticipated in the dark-matter subsystems of galaxies, the second one ($i = 3$) is sufficiently close to the King (1962) mass distribution usually anticipated in globular clusters, whereas the last one, in view of the recent results (more precisely, density dependence of $\rho(r) \propto r^{-4}$ in outer parts - e. g. Dehnen, 1993), might be applicable to the ellipsoids of elliptical galaxies, i. e. to the bulges of spiral ones.

It should be emphasized that formula (1) has an advantage that the corresponding surface density is very easily obtainable analytically. The concrete expressions can be found in the mentioned paper (Ninković, 1998). More clearly, the expressions of both kinds - those concerning the space density and those concerning the surface one - contain algebraic functions only. This, however, need not be the case with other quantities of interest: the potential, the mean velocity squares, etc. The exception is the classical Schuster case ($i = 5$) where, if one remains in the framework of the isotropic velocity distribution, all these functions are algebraic and easily obtainable. Therefore, of interest might be alternative formulae where one has, for instance, for the potential an expression consisting of algebraic functions and which can be easily related with family (1). Such a case is with a family of potential formulae considered by Kuzmin and his disciples. Here the corresponding potential formula is given in its most general case, i. e.

$$\Pi(r) = \frac{G\kappa\mathcal{M}}{a + [b^2 + (\kappa r)^2]^{1/2}}\,, \tag{2}$$

where r is, as earlier, the distance to the centre of the stellar system under study, Π is its potential, G is the gravitation constant, $\mathcal{M}$ is the total mass of the system, a, b and κ are the system parameters, the former two are the scale lengths, and the latter one is dimensionless. The Schuster classical model is a special case of (2) when $a = 0$, $\kappa = 1$. If a is different from zero, the density dependence in the outer parts is also $\rho(r) \propto r^{-4}$ (for details Ninković, 2001 and the references therein).

In the same paper (Ninković, 2001) one finds the common case of (1) and (2) for $i = 4$ (in (1)) and $\kappa = 1$ (in (2)). The use of two additional dimensionless parameters in (2) (ratio a/b and κ) offers new possibilities. For instance, the interesting special case of (1) - $i = 4$ - yields a nearly constant density within the central part, without a cusp (about this term e. g. Dehnen, 1993). However, if the observations indicate a cuspy mass distribution for a stellar system, then the special case $i = 4$ of (1) requires amendments in order to improve the fit. One way is to introduce an additional subsystem (say, a nucleus) or a formula containing more parameters, but realistic, i. e. interpreting the cusp not by means of a central singularity, but through a prominent maximum at the centre. This is possible by using (2) allowing κ to be arbitrarily large, but, nevertheless, finite. Then, as some preliminary studies of the present author show, the maximum on the circular-velocity curve will be realistic, i. e. it will occur for some value of the ratio r/b greater than zero. On the other hand, for some models having central singularity this maximum can reach the centre, itself, or even cease to exist for steeper density slopes. A typical example is the family considered, for instance, by Dehnen (1993).

A few words may be said concerning the special case of (1) $i = 3$. It has been already compared to the King (1962) case. It should be said that, if the space density is treated correctly, one also obtains a critical value where the surface one vanishes regardless of the existence of the discontinuity at the boundary mentioned above. As indicated by the present author (Ninković, 1998), this case has already been applied to star clusters, the field of very frequent application of King's (1962) density formula.

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