



Gaussian processes for

# long-term observed continuum and broad line light curves of E1821+643

A. Kovačević<sup>1</sup>, L. Č. Popović<sup>2</sup>, A. I. Shapovalova<sup>3</sup> & D. Ilić<sup>1</sup>

<sup>1</sup>Department of Astronomy, Faculty of Mathematics, University of Belgrade, Serbia

<sup>2</sup> Astronomical Observatory Belgrade, Serbia,

<sup>3</sup> Special Astrophysical Observatory of the RAS, Russia

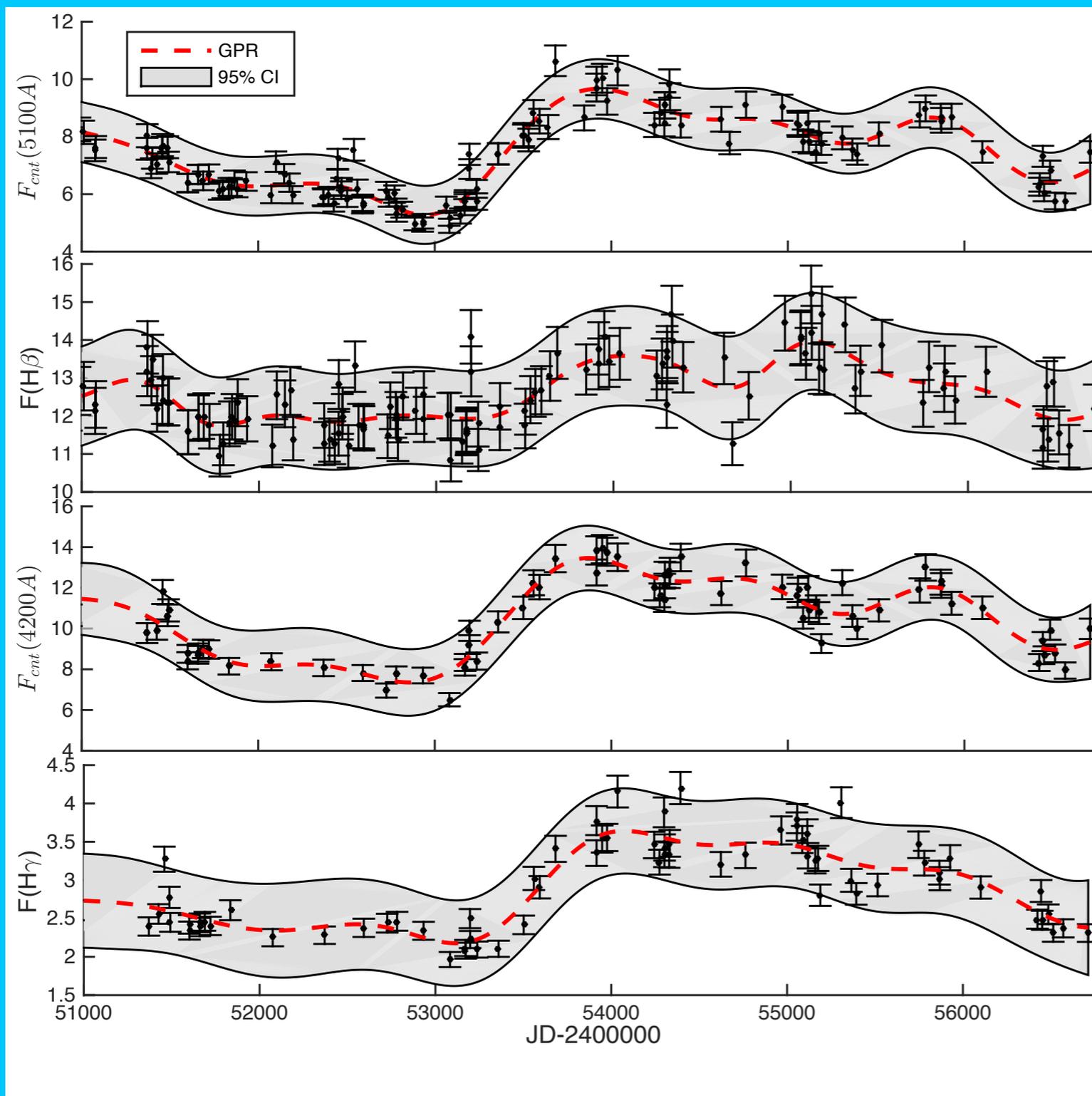


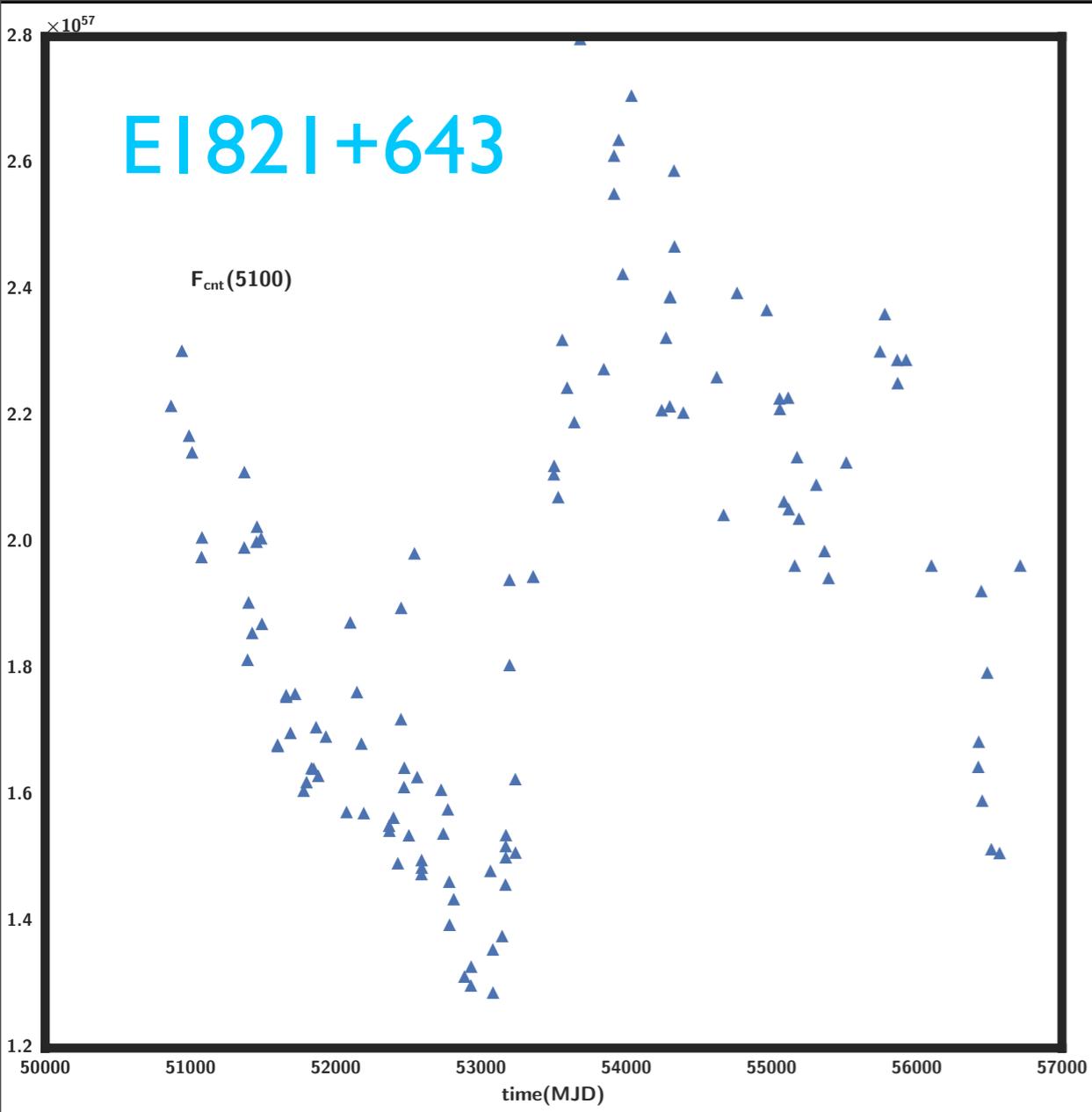
Fig. 12.— Comparison between the GPR generated (dashed line) and observed light curves (circles with errorbars) of the continuum at 5100Å, H $\beta$ , the continuum at 4200Å, and H $\gamma$  (from top to bottom). The shaded band represents 95% confidence interval (CI) for the GPR predicted curve.

**Shapovalova**  
**@ al+16**  
**Task: Periodicity?**  
**P1 ~4500 days**  
**P2 ~2000 days**  
**P3 ~1300-1400 days**

Table 8. Detected flare-like events in photometric data.

N	UT-Date	MJD 2400000	Amplitude [m]	I <sub>max</sub> /I <sub>min</sub>	maxJD(i+1)-maxJD(i) [days]
1	2005Nov28	53703	0.54	1.6	1066 (2-1)
2	2008Oct29	54769	0.10	1.1	1030 (3-2)
3	2011Aug26	55799	0.17	1.17	1002 (4-3)
4	2014May23	56801	0.10	1.1	

Note. — Col.(1): number of flare-like event; Col.(2): UT-date; Col.(3): Modified Julian Date; Col. (4): approximate amplitude between maximum and minimum (in magnitudes) for a flare-like event; Col.(5): the ratio of the maximum to minimum intensity; Col.(6): the difference in days between the two relevant events (in parentheses).



**Nipoti & Binney +05**  
AGN Variability is large

^

Luminosity  $L(t) \gg 0, \forall t$



The natural variable to model is  $I(t)=\ln(L)$   
and

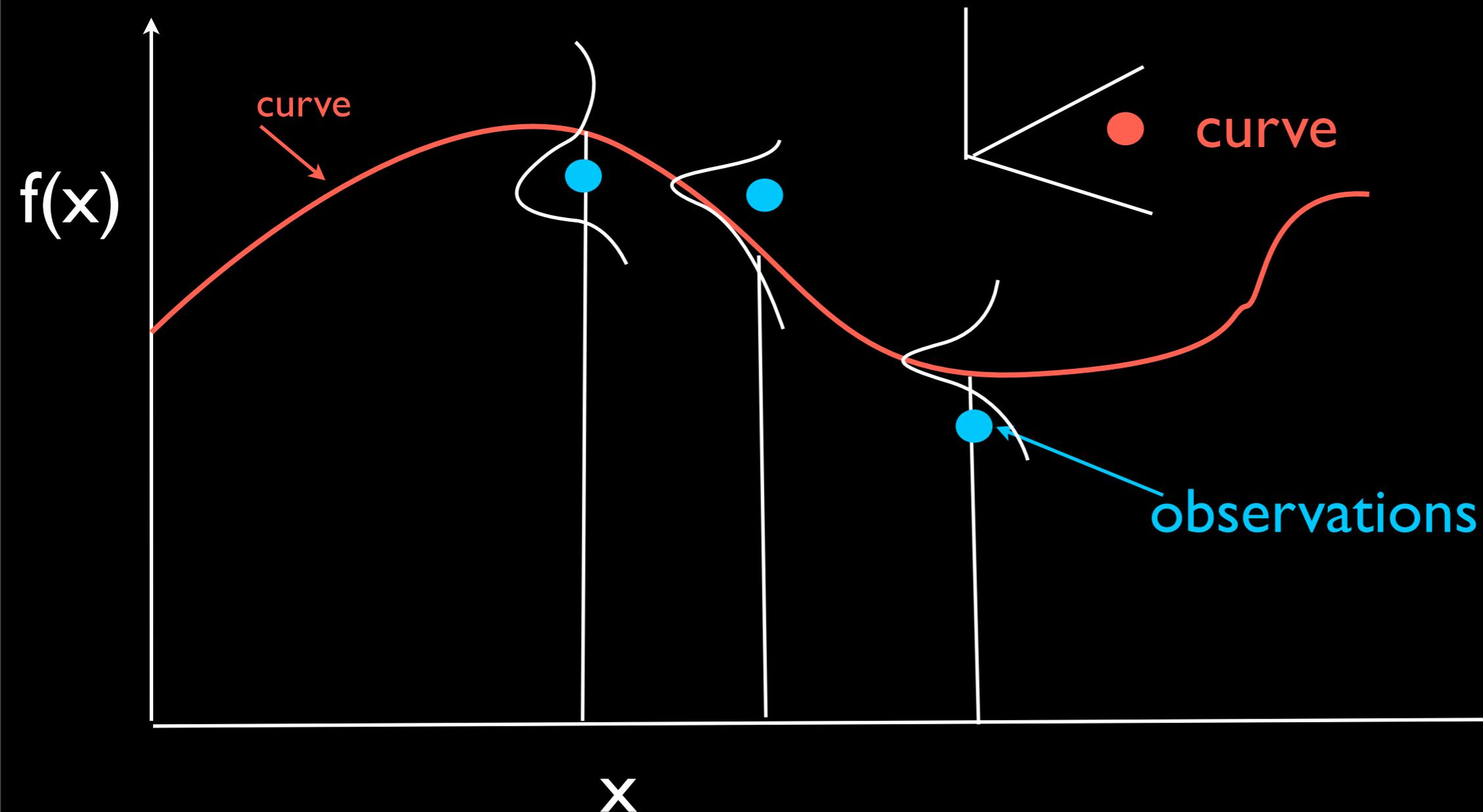
Default assumption is

$I(t) \in \text{Gaussian Processes (GP)}$

# What is GP?

multiple perspectives for understanding a GP:

1. As an *infinite dimensional Gaussian random variable* with a specified co-variance structure. This view tells you what it is and what its properties are. But this doesn't explain how people make use of Gaussian process models.
2. *Weight space view*: The predictions of a Gaussian process are weighted averages of the training target values. However, the weighting scheme is not simple as in the case of locally weighted regression. For more information on this see [Sollich's Paper](#).
3. *The function space perspective*: This is a tricky one for the uninitiated. A Gaussian process is a distribution over functions. Finite dimensional Gaussians are distributions over finite dimensional vectors. Infinite dimensional Gaussians (Gaussian Processes) are distributions over infinite dimensional vectors (or equivalently, functions).



# What is GP (cont'd)?

## Gaussian distribution

\* is a distribution over **vectors**

\*specified by mean and a covariance  
(vectors)  $x \sim G(\mu, \Sigma)$

\*the position of the random variables  
 $x_i$  plays the role of the index

## Gaussian process

\* is a distribution over **functions**

\*specified by mean function and a  
covariance function  $f \sim GP(m, k)$

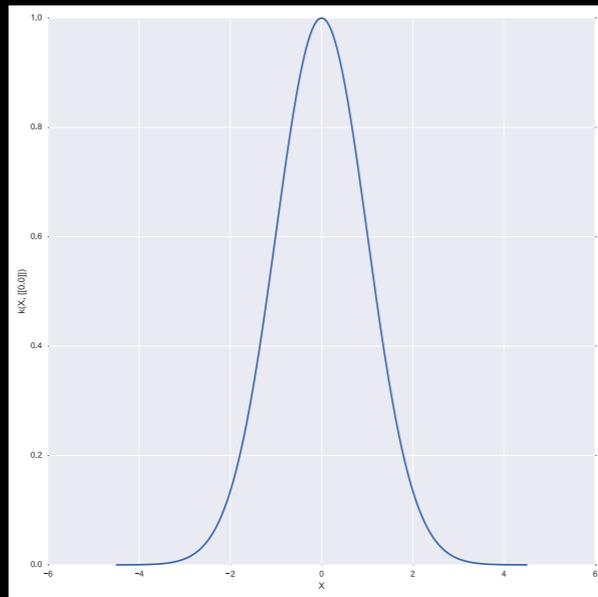
\*the argument  $x$  of the random  
function  $f(x)$  plays the role of the  
index

# Defining GP

by mean function and a covariance function  $f \sim \text{GP}(m, k)$

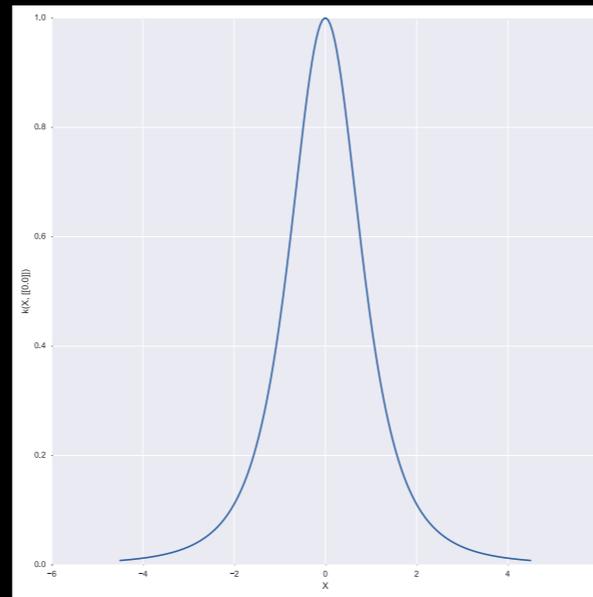
mean function is defined  $m \sim 0$   
different covariance functions

## RBF kernel



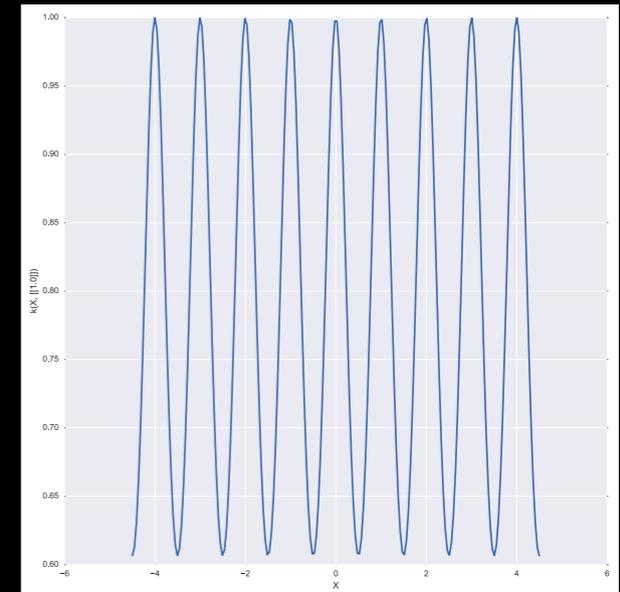
$$K_{\text{SE}}(x, x') = \exp\left(-\frac{\|d\|^2}{2l^2}\right)$$

## RQ kernel



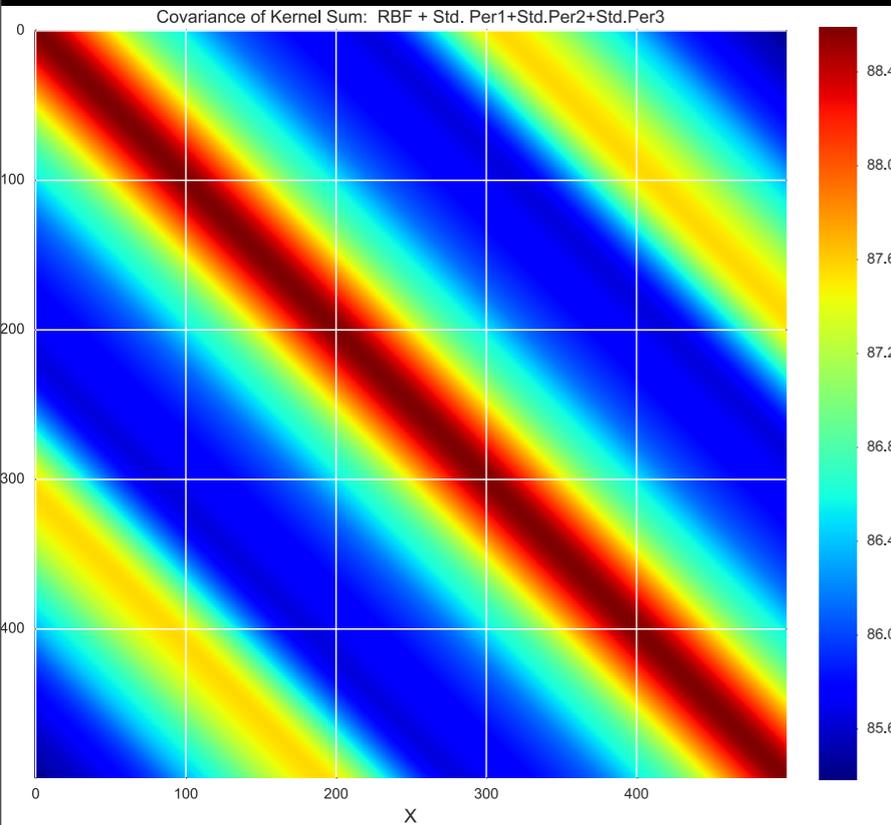
$$K_{\text{RQ}}(x, x') = (1 + |d|^2)^{-\alpha}, \quad \alpha \geq 0$$

## Std. Periodic kernel

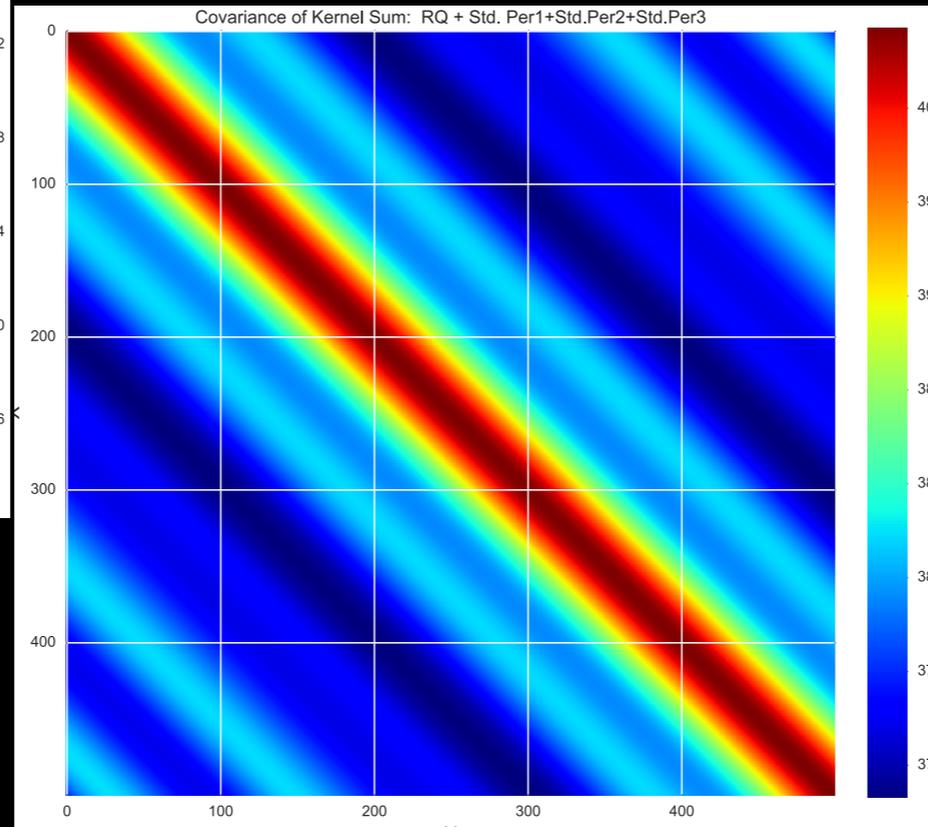


$$K_{\text{P}}(x, x') = \exp\left(-\frac{2 \sin^2\left(\frac{d}{2}\right)}{l^2}\right)$$

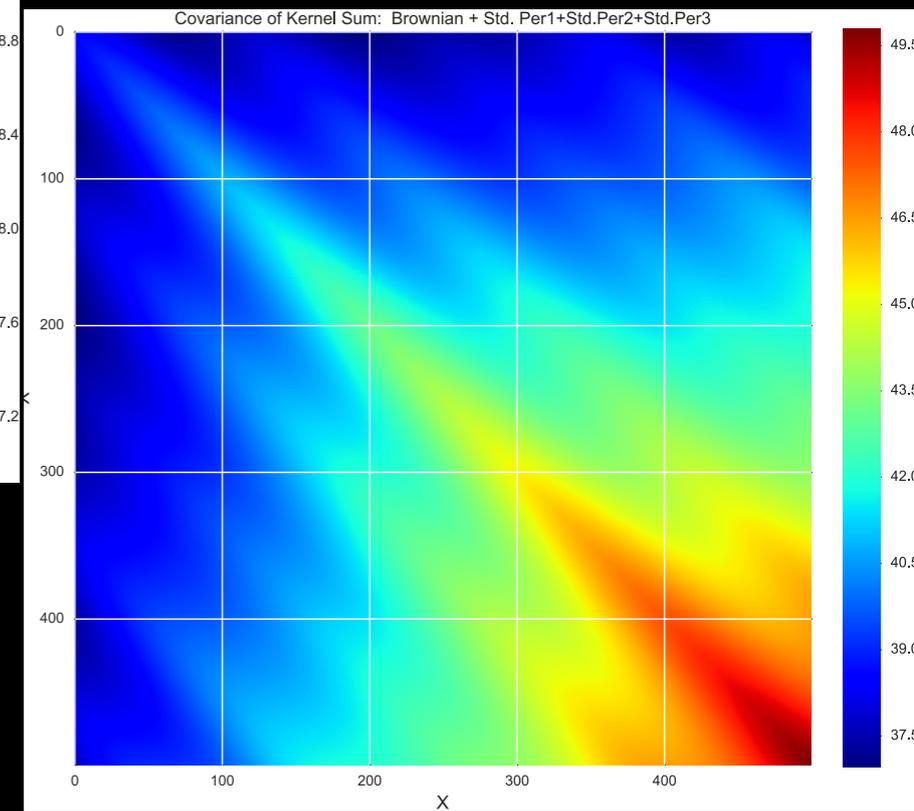
# COVARIANCE



**RBF+Std.Per(4500)+  
Std.per(2000)+Std.per(1400)**



**RQ+Std.Per(4500)+  
Std.per(2000)+Std.per(1400)**



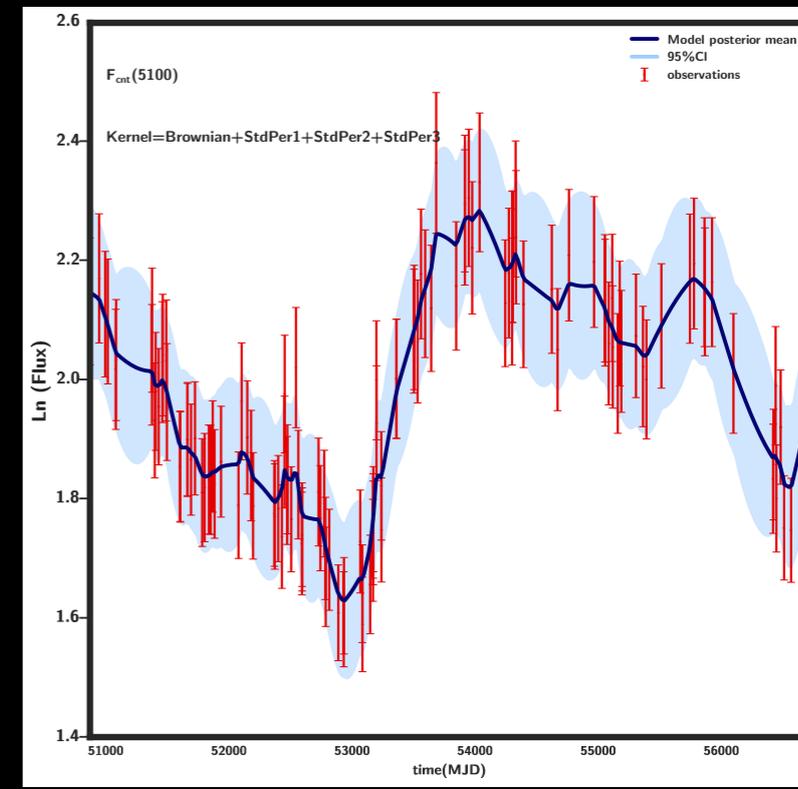
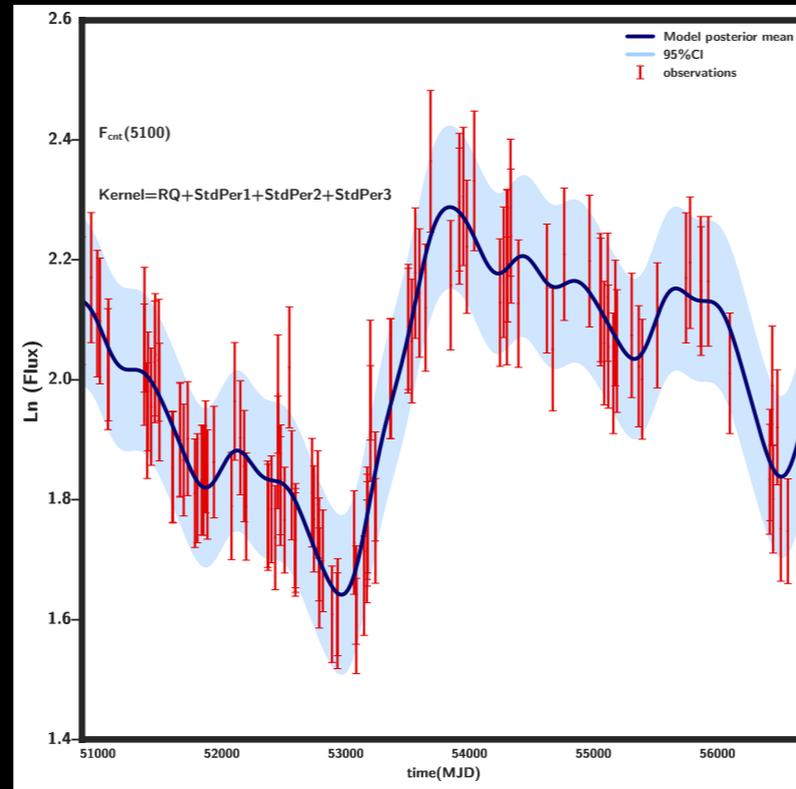
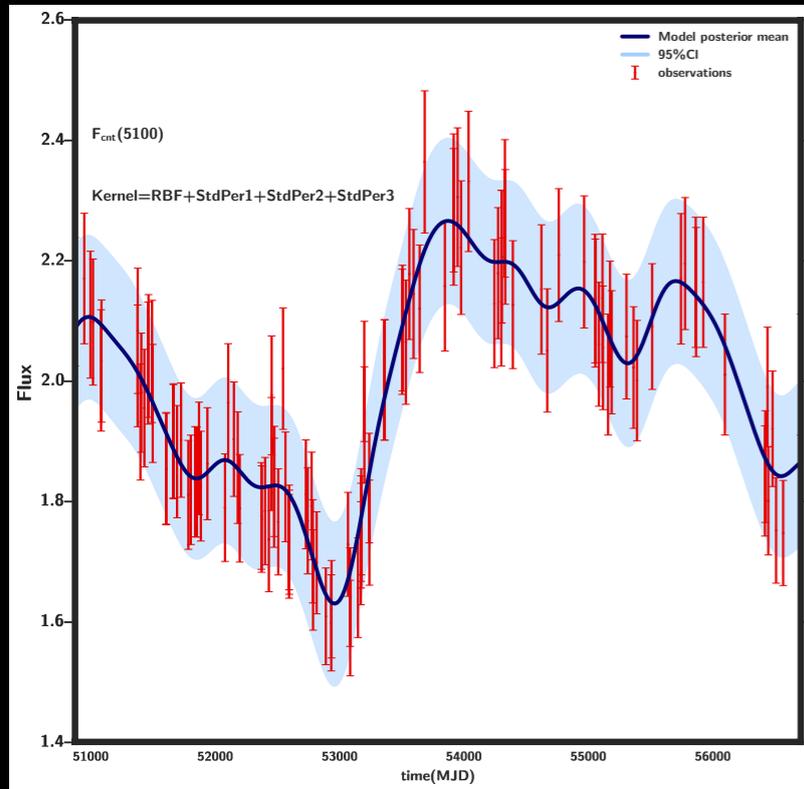
**Brownian+Std.Per(4500)+  
Std.per(2000)+Std.per(1400)**

# Continuum 5200

RBF+StdPer1+StdPer2+StdPer3

RQ+StdPer1+StdPer2+StdPer3

Brownian+StdPer1+StdPer2+StdPer3



**ML - I32**

**PI ~4800days**

**P2 ~2300days**

**P3~1600 days**

**RMSE 0.062**

**S~1.00**

**ML - I33**

**PI ~3900days**

**P2 ~2300days**

**P3~1200 days**

**RMSE 0.062**

**S~1.02**

**ML - I27**

**PI ~3800days**

**P2 ~1800days**

**RMSE 0.047**

**S~1.02**

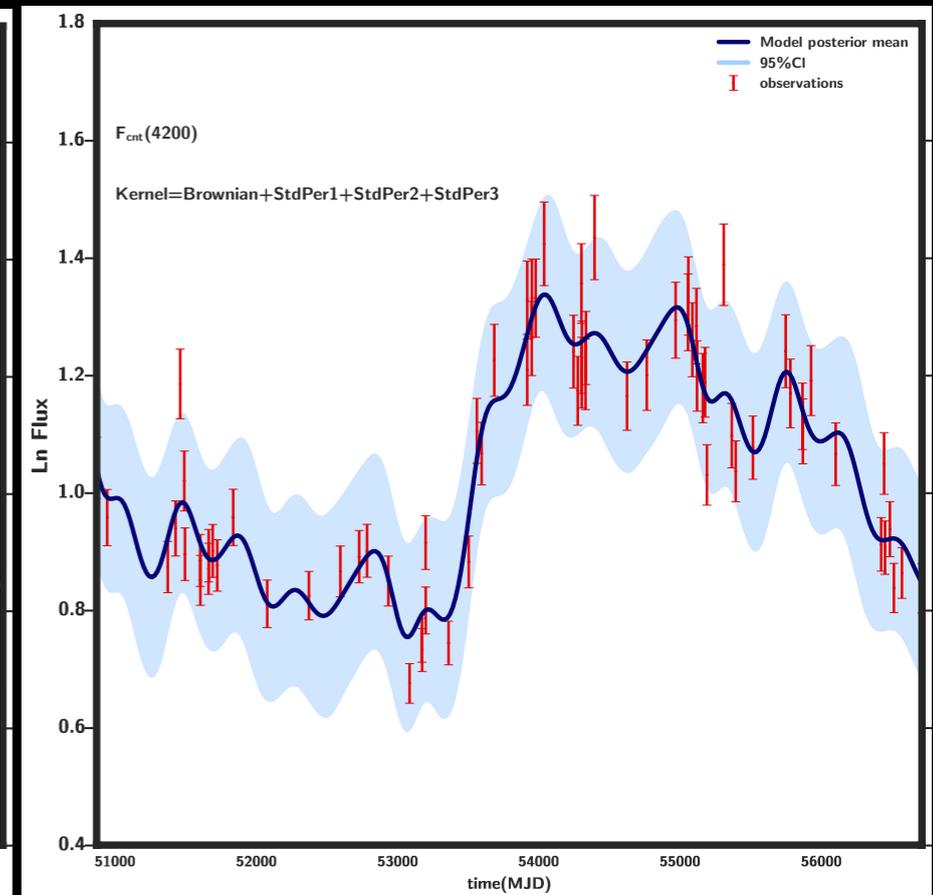
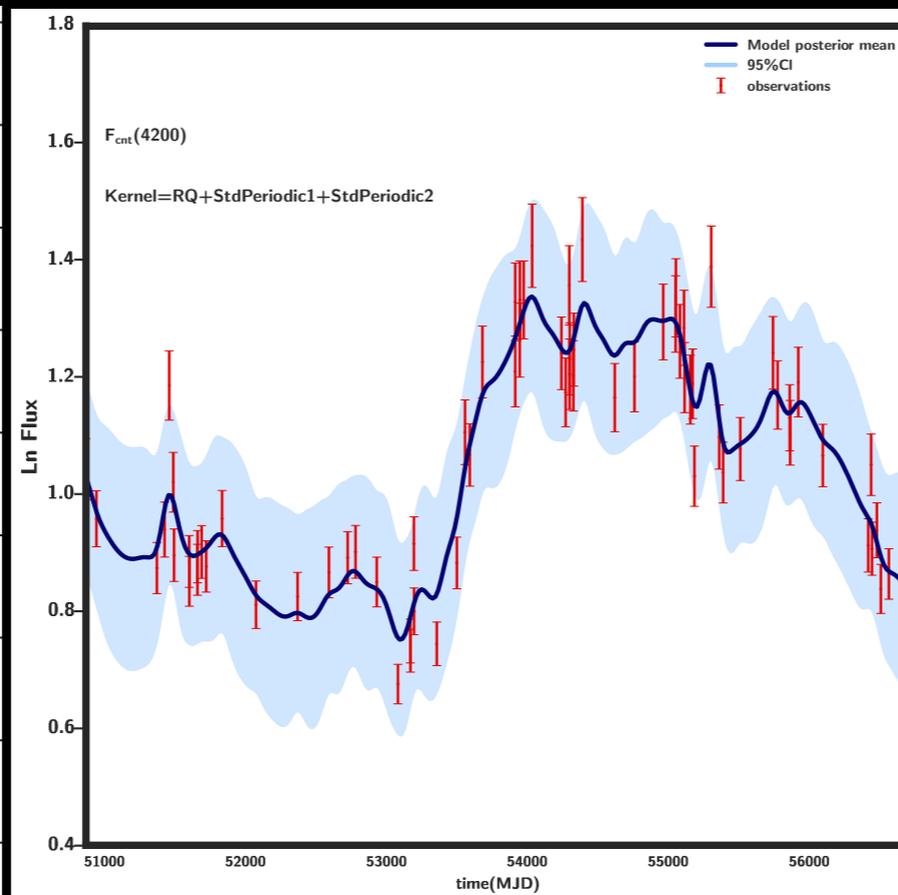
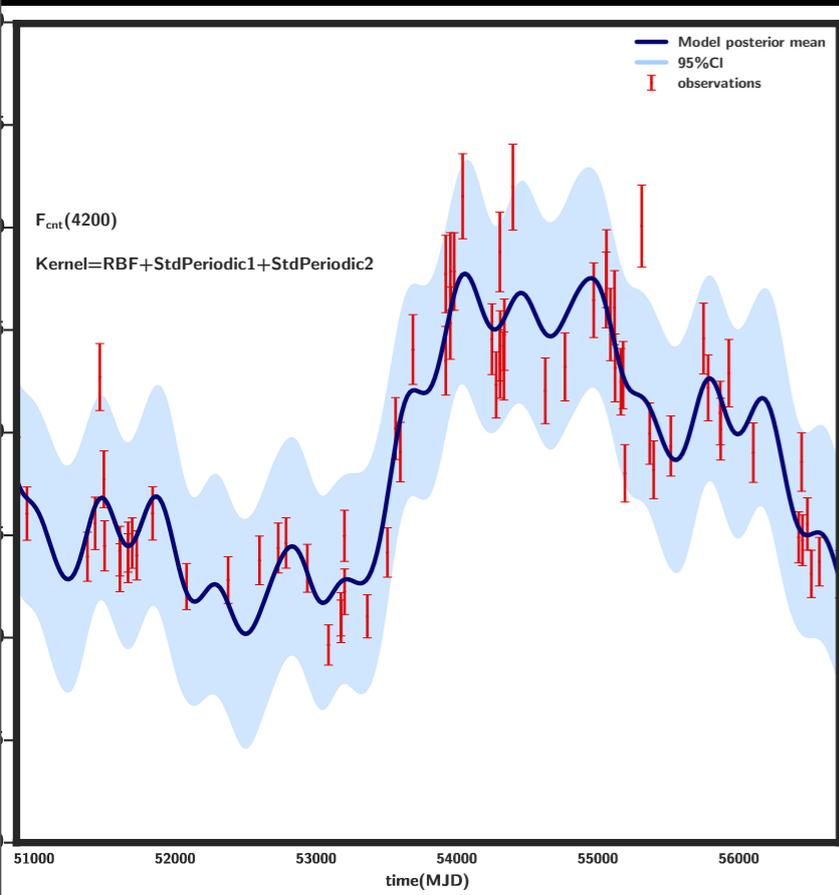


# continuum 4200

RBF+StdPer1+StdPer2

RQ+StdPer1+StdPer2

Brownian+StdPer1+StdPer2+StdPer3



**ML -70**

**PI ~5000days**

**P2 ~2200days**

**RMSE 0.03**

**S~1.00**

**ML -62**

**PI ~5000days**

**P2 ~2200days**

**RMSE 0.06**

**S~1.1**

**ML -63**

**PI ~5000days**

**P2 ~2100days**

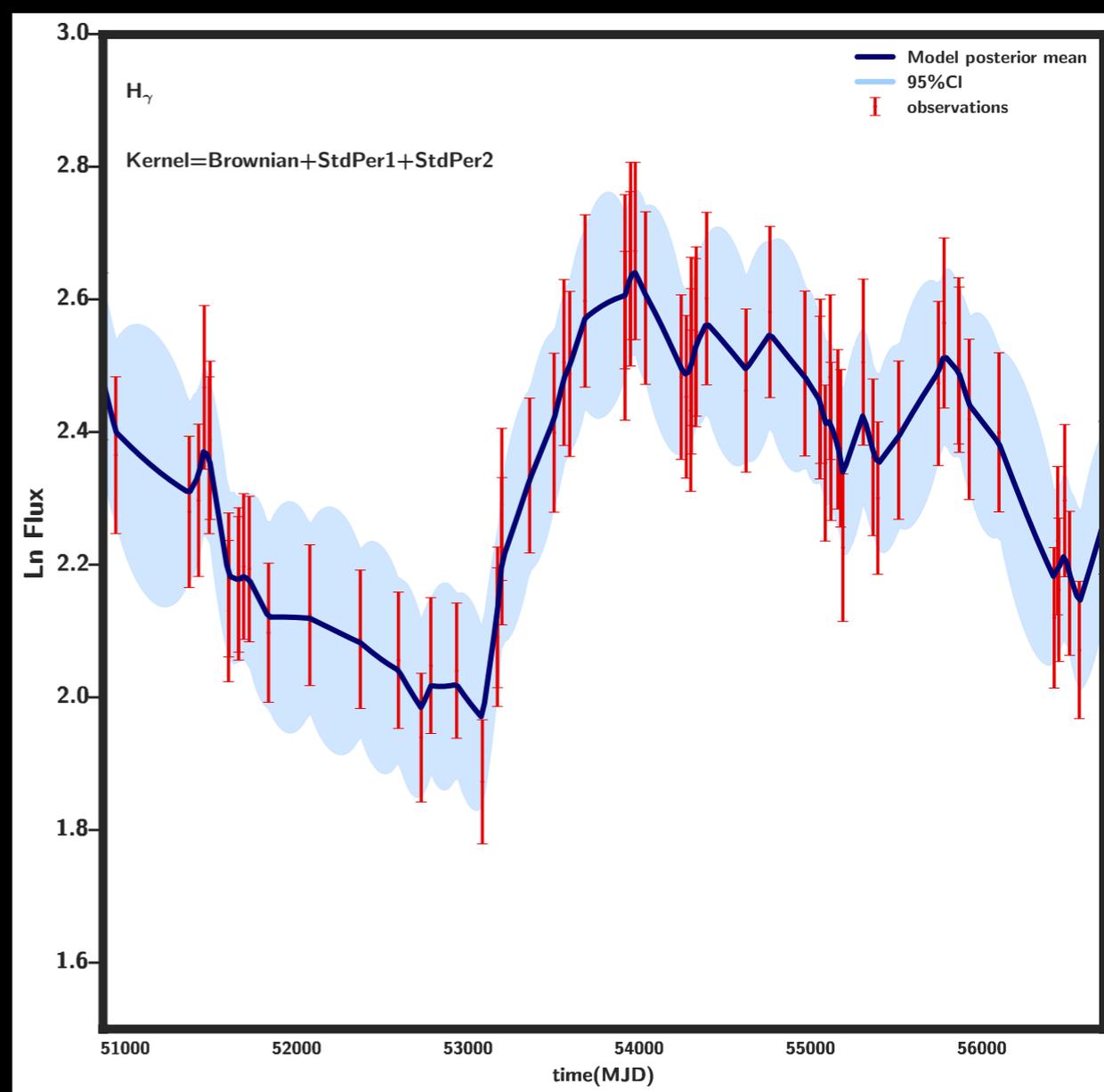
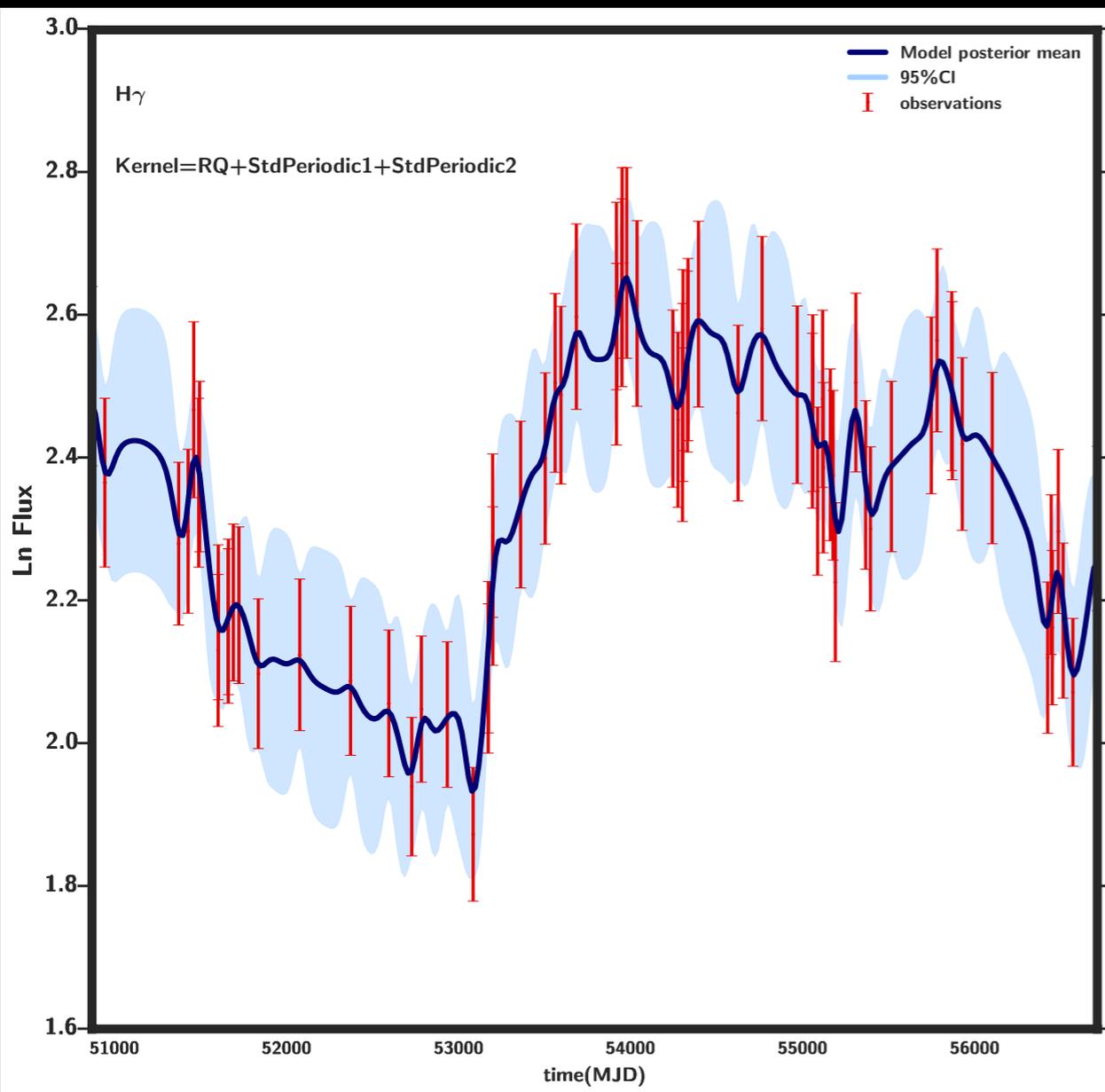
**P3 ~1500 days**

**RMSE 0.07**

**S~1.13**

## RQ+StdPer1+StdPer2

## Brownian+StdPer1+StdPer2+StdPer3



**ML -70**  
**PI ~4800days**  
**P2 ~2500days**

**RMSE 0.03**  
**S~1.85**

**ML -64**  
**PI ~4000days**  
**P2 ~1900days**

**RMSE 0.04**  
**S~1.44**

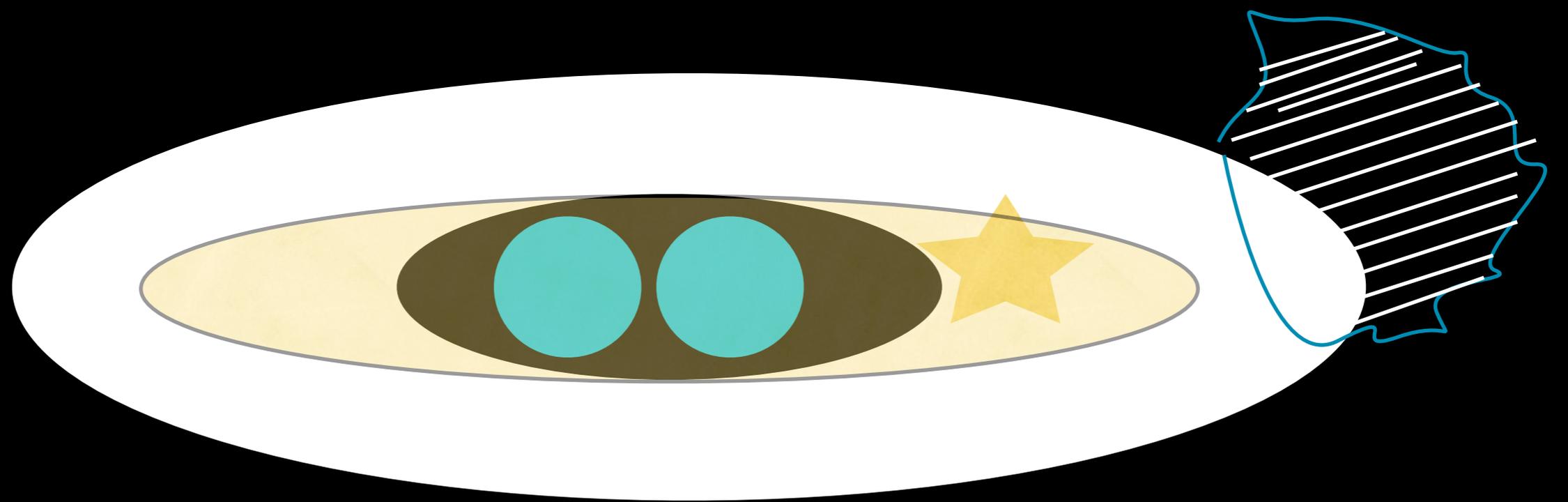
# Key points

Mixture of aperiodic (RBF,RQ,Brown) and periodic (Standard Periodic) kernel models indicates high periodicity ratio  $S \sim 1$  and presence of periodic signals in both continuum and Hgamma emission line

Two periodic signals P1~4800 days and P2 ~2000 days are persistent in continuum 5200 and 4200 and H $\gamma$  emission line.

The third periodic signal P3~1400 days are not persistent: it is very weakly presented in both continuums and completely lost in Hgamma line. Indicating that information about it has been erased by processes originating at the distances of Hgamma. Because of this, 1400 days signal can be caused by some process like flares within the disk, originating closer to the BH.

Any information about periodic signals are completely erased from Hbeta line due to large noise. Since the time lag Hbeta > time lag Hgamma, it implies that the Hbeta line originates at larger radii than Hgamma, so at the distance of Hbeta line can exist some violent processes destroying all periodic signals



# Acknowledgments

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