The radio lines of hydrogen

Gillian Peach

**Department of Physics and Astronomy,
University College London**

- Low density plasmas; Historical Background
- Spectral Line Broadening; Baranger Theory and the Impact Approximation
- Results

Introduction

• Radio lines are observed in regions with densities
$10^3 \leq N_e \leq 10^4$ and $T_e \simeq 10^4$K.

• In 1959, Kardashev first predicted that these lines could be observed and gave some estimates for the Doppler and Stark broadening to be expected. These proved to be inconsistent with observation.

• Hans Griem (1967) realised that under these conditions both proton and electron collisions should be treated using the impact theory of line broadening and did obtain consistent results. Essentially the same conclusions were reached by me in (1972). Many observations of radio lines emitted from various sources have been published since then.

• The book by Gordon and Sorochenko (2009) gives a comprehensive review of observations and theory.

Reasons for reexamination of the line broadening theory

• In 2000, Morley Bell published observations of radio lines emitted at frequencies around 6 GHz and 17.6 GHz by Orion A and W51. More details have been given in a (2011) paper.

The results for 17.6 GHz did not present any surprises but the ones at 6 GHz did.

Transitions $n' = n + \Delta n \rightarrow n$ observed were: $(n, \Delta n)$ = (102,1), (129,2), (147,3), (174,5), (184,6), (194,7), (202,8), (210,9), (217,10), (224,11), (230,12), (236,13), (241,14), (247,15), (252,16), (257,17), (261,18), ((266,19), (270,20), (274,21), (278,22), (282,23), (286,24) and (289,25).

Lines above $(n, \Delta n)$ = (202,8) showed unexpected narrowing. Hence the recent reexamination of line broadening theory by several authors, Oks (2004), Griem (2005), Watson (2006) and Hey (2012 and previous papers); private communication (2013).

In previous calculations complete profiles for the lines have not been calculated.
The purpose of the present calculations is to obtain complete profiles and then extract the widths.

- Baranger Impact Theory

Baranger (1958) developed impact theory for the case of overlapping lines, and for an isolated line this reduces to the well-known expression for the full-half width of the Lorentzian profile for the transition $i \rightarrow f$ given by

$$W = \left[Nv \left(\sigma_i(\text{in}) + \sigma_f(\text{in}) + \int \mathrm{d}\Omega |f_i(\Omega) - f_f(\Omega)|^2 \right) \right]_{av}$$

In the case of hydrogen we have to consider all the overlapping components in the $(n, \Delta n)$ transition. For low values of n the third term dominates, but for a fixed value of Δn as n increases the elastic scattering amplitudes f_i and f_f coherently cancel and only the inelastic cross sections σ_i(in) and σ_f(in) contribute. For overlapping lines in hydrogen the elastic scattering terms should be interpreted as including all transitions for which $\Delta E = 0$.

• Overlapping line profiles

The formal expression is given by

$$L(\omega) = \frac{1}{\pi}\mathcal{R}\sum << n_i(n_f)^*||\delta||n'_i(n'_f)^* >>$$
$$\times << n'_i(n'_f)^*||[\mathbf{h} - \mathsf{i}(\omega - h_0/\hbar)]^{-1}||n_i(n_f)^* >> .$$

This has been used to calculate the full profile. The operator **h** gives rise to the line broadening. For each frequency $(n_i n_f)\times(n_i n_f)$ must be inverted.

Only the dipole interactions between the electrons/protons and the emitting hydrogen atom are included. Treatment of the collisions is semi-classical using impact parameter methods developed by Seaton (1962). Proton impact is included but proves to be unimportant.

References

M. Baranger, Phys. Rev. **112** 855 (1958)

M.B. Bell *et al*, Publ. Astron. Soc. Pac. **112** 1236 (2000)

M.B. Bell *et al*, Astrophys. Space. Sci **333** 377 (2011)

M.A. Gordon and R.L. Sorochenko, 'Radio Recombination Lines', Springer, NY (2009)

H.R. Griem, Astrophys. J. **148** 547 (1967); **620** L133 (2005)

J.D. Hey, Private Communication and references in J. Phys. B: At. Mol. Opt. Phys. **39** L291 (2012)

E. Oks, Astrophys. J. **609** L25 (2004)

G. Peach, Astrophys. Lett. **10** 129 (1972)

M.J. Seaton, Proc. Phys. Soc. **79** 1105 (1962)

J.K.G. Watson, J. Phys. B: At. Mol. Opt. Phys. **39** 1889 (2006)

Results

The first table presents results to be compared with those of Watson (2006) and good agreement is obtained. There is no evidence of narrowing for $n > 202$ as the observations of Bell suggests.

Hey (private communication) has carried out a new analysis of Bell's data taking into account relative intensities of the lines as well as their widths and these are shown in the second and third tables.

The widths of the radio lines at 6 GHz

Widths are tabulated for transitions $(n, \Delta n)$ relative to that for (102,1) with T $= 1.2530 \times 10^4$ K.

Line	Ratio	Line	Ratio
(102,1)	1.000	(241,14)	33.12
(129,2)	2.569	(247,15)	36.91
(147,3)	4.309	(252,16)	40.40
(174,5)	8.462	(257,17)	44.15
(186,6)	10.60	(261,18)	47.47
(194,7)	13.17	(266,19)	51.76
(202,8)	15.56	(270,20)	55.56
(210,9)	18.31	(274,21)	59.59
(217,10)	21.02	(278,22)	63.86
(224,11)	24.07	(282,23)	68.39
(230,12)	26.98	(286,24)	73.17
(236,13)	30.18	(289,25)	77.22

Orion A. Analysis by Hey (Private communication) of Bell's observations for FWHM widths W(V) in MHz, and the impact widths used, W(I).

Line	$N_e(10^3\text{cm}^{-3})$	T_e (10^4 K)	W(V)	W(I)
(102,1)	7.75	1.300	0.547	0.095
(129,2)	6.12	1.296	0.591	0.186
(147,3)	4.34	1.276	0.610	0.216
(174,5)	7.36	1.301	0.945	0.696
(184,6)	4.56	1.253	0.827	0.537
(202,8)	3.59	1.224	0.870	0.612
(252,16)	5.84	1.228	0.617	0.244
(274,21)	4.16	1.146	0.609	0.254

W51. Analysis by Hey (Private communication) of Bell's observations for FWHM widths W(V) in MHz, and the impact widths used, W(I).

Line	$N_e(10^3\text{cm}^{-3})$	T_e (10^4 K)	W(H)	W(I)
(102,1)	2.62	2.094	0.641	0.030
(129,2)	2.34	2.031	0.643	0.066
(147,3)	1.53	1.994	0.645	0.070
(174,5)	2.94	1.994	0.748	0.255
(184,6)	2.37	1.933	0.745	0.256
(194,7)	2.62	1.952	0.800	0.345
(202,8)	2.36	1.948	0.815	0.363
(210,9)	1.51	1.914	0.749	0.272
(224,11)	1.29	1.920	0.767	0.300