Line shapes and intensities in fluctuating plasmas

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Outline

1. Introduction
2. Theory of stochastic processes
3. Application to Stark profiles
4. Application to the population kinetics of atoms in a turbulent plasma
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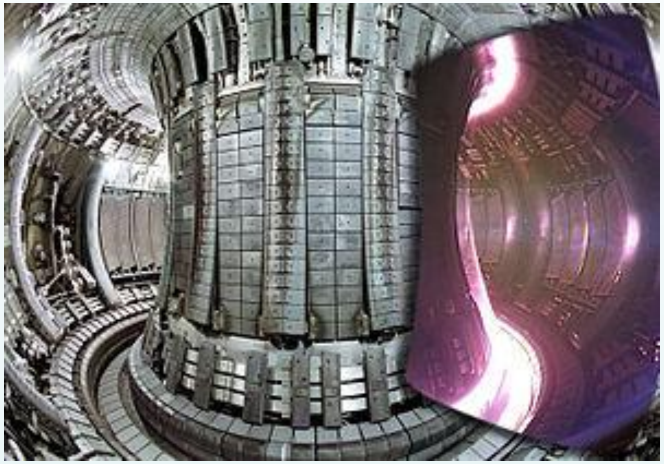
Modeling of radiative properties of plasmas

-Radiative properties for plasma codes:
Fast and reasonably accurate
(edge codes, astrophysics)
-Radiative transfer, plasma diagnostic
-Stark and Doppler broadening
-Line intensity ratios in turbulent plasmas



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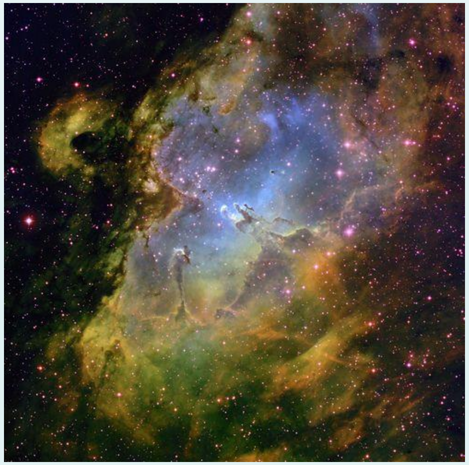
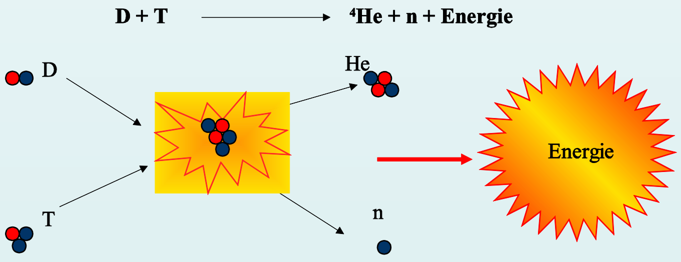


Astrophysics

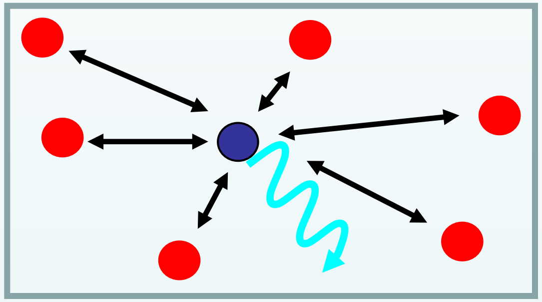


Fusion

Modeling of radiative properties of plasmas

Particles

- Statistical mechanics
- Ab initio calculation



Simulation of a large number of particles, coupled to a numerical integration of the Schrödinger equation

Stochastic approach:

-Applied to a fluctuating plasma field:

Electric microfield, density or temperature

-Used today for line shapes, but also recently for population kinetics, neutral transport in a turbulent plasma

Stochastic process

Used by the Model Microfield Method (MMM) for Stark effect (Frisch and Brissaud, Stehlé)

-What is the behaviour of the MMM line in a near impact regime

(in the center of mass frame)?

- Re-evaluation and tentative improvement of the process

Two complementary approaches used

Analytical:
Complex calculations, but
fast evaluations

Numerical:
Monte Carlo simulation
Easy to implement even for
complex probabilities, but
slow numerical evaluations

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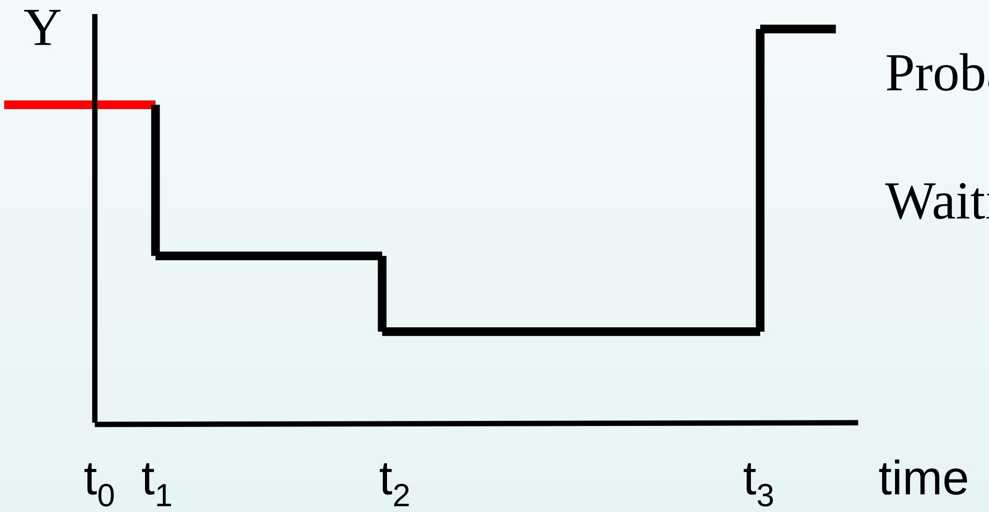
Stationnary renewal process

The plasma field Y(t) is assumed to be stepwise constant



Probability P(Y) of having a field Y

Waiting time distribution WTD(t,Y)

The measure of a radiative property starts at a time t=0:

This time is generally not a jumping time

We need to distinguish between the first and the following steps: stationnarity conditions

Renewal process and stationnarity conditions

Stationnarity conditions require different probability density functions (PDF) for the first than for the following steps

	Two PDF for the field	Two WTD, waiting time distribution:
First step	P(Y)	$v_Y(t)$
Next steps	Q(Y)	$w_Y(t)$

But we have stationnarity conditions

$$\begin{cases} Q(Y) = \dfrac{v_Y(t=0)P(Y)}{\left\langle v_Y(t=0)\right\rangle_S}, \\ w_Y(t) = -\dfrac{\dot{v}_Y(t)}{v_Y(t=0)} \end{cases}$$

Only P(Y) and $v_Y(t)$ are constrained by plasma statistical properties

Exact solution of the stochastic equation

Stochastic evolution equation :

$$\begin{cases} \frac{d}{dt} X(t) = M(Y(t))\, X(t), \\ X(t=0) = X_0 \end{cases}$$

X(t) may be an atomic population **or** an atomic evolution operator (Stark effect).

- An exact solution of the stochastic equation is obtained by using a Laplace transform, it depends on P(Y) and $v_Y(t)$

-The PDF P(Y) is measured or calculated

-We constrain the WTD $v_Y(t)$ with a plasma dynamical property : e.g. correlation function of Y(t)

-Note this usually leaves many possible choices for $v_Y(t)$

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Stark broadening: the line shape

Fluctuating parameter: the electric microfield $Y = \vec{E}$

Fourier transform of the dipole autocorrelation function

$$L(\omega) = \frac{1}{\pi} \mathrm{Re} \int_0^\infty C(t) e^{i\omega t} dt$$

Time of interest

$$\tau_i \approx 1/\Delta\omega_{1/2}$$

$$C(t) = \mathrm{Tr} \left\langle \rho\, \vec{d}(0)\, U^+(t) \vec{d}(0)\, U(t) \right\rangle_{av}$$

The evolution operator obeys to the Schrödinger equation

$$i\hbar \frac{dU}{dt}(t) = \left(H_0 + V(t)\right) U(t)$$

$$V(t) = -\vec{d}.\vec{E}(t)$$

Static and impact approximations

Two limiting regimes

<u>Static</u>:

$$\tau_i \ll \tau_c \quad\Rightarrow\quad \vec{E}(t) \approx \vec{\mathrm{E}}(0)$$

$\tau_i \approx 1/\Delta\omega$ Time of interest

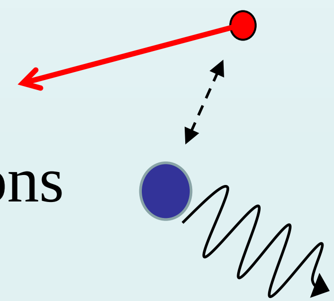
$\tau_c \approx r_0/\mathrm{v}$ Collision time

Only the PDF P(E) is needed

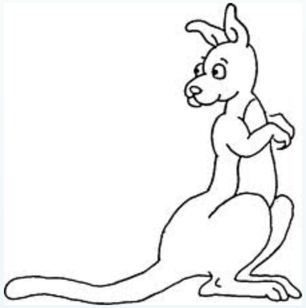
<u>Impact</u>:

$$\tau_i \gg \tau_c \quad\Rightarrow$$

Binary collisions (electrons)



The Kangaroo process (KP)



Brissaud et Frisch 1971, Seidel 1977, Frerichs 1989
Stehlé 1994, 1999, 2010
Markovian process (no memory)

$$\nu(t|E) = \nu(E)\exp(-\nu(E)\,t)$$

Kangaroo Process

$\nu(E)$ is the microfield dependent jumping frequency

Two statistical properties of the microfield are used:
-PDF P(E)
-Plasma microfield correlation function $\Gamma_{plasma} = \langle \vec{E}(0)\vec{E}(t) \rangle$

Calculations for ions only

<u>Ion dynamics effects are intermediate between static and impact</u>

-Many body dynamic effect and quantum problem

In the following, we compare our Kangaroo Process (KP) calculations to ab initio simulations for **ions only**

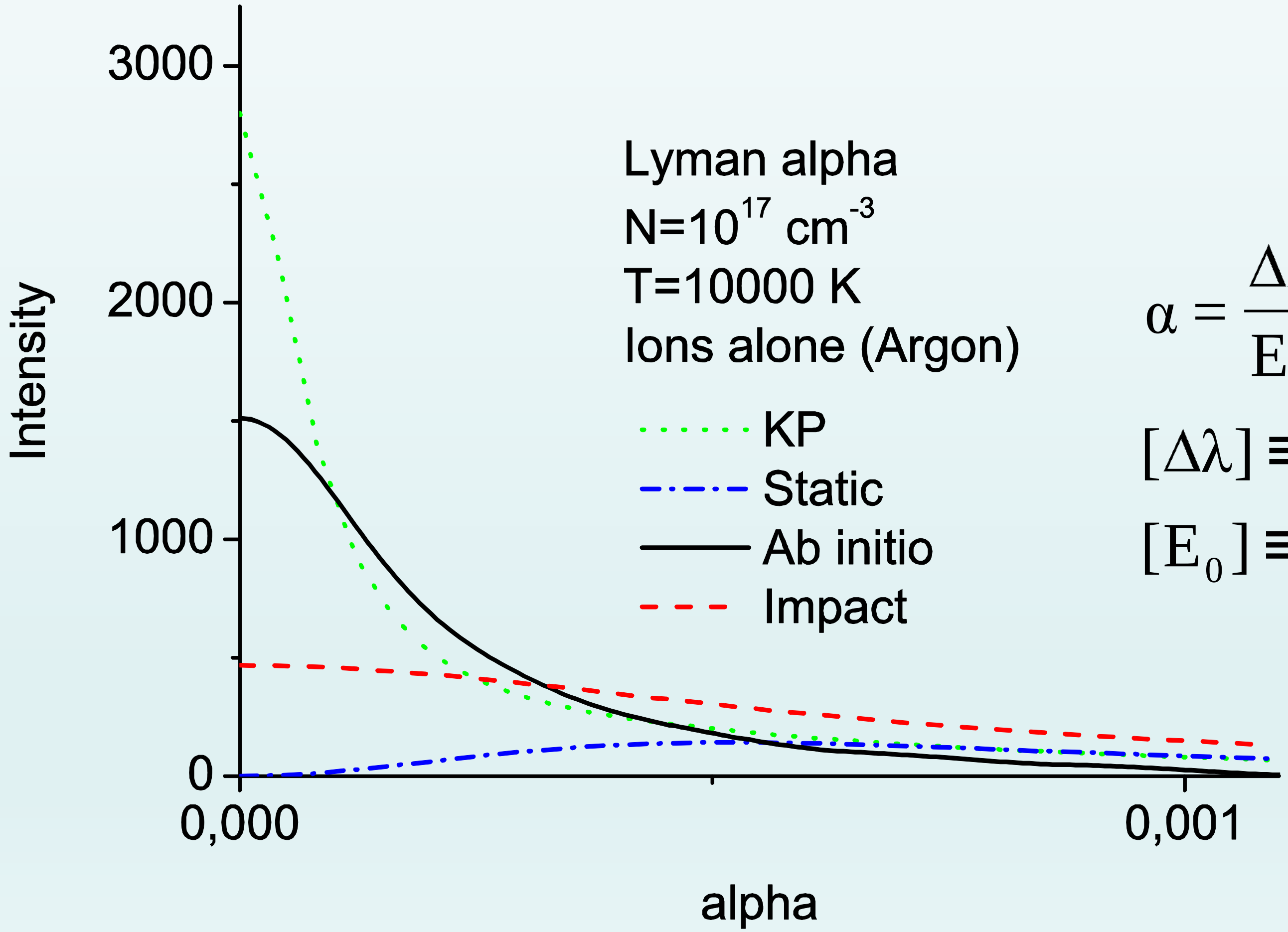
! Modeling test. Cannot be compared to experiments:

No electrons, no fine structure, no Doppler

Lyman α with ions alone, N=10^{17} cm^{-3}

- Reference : ab initio simulation

- Ion dynamics effects $\tau_c \approx \tau_i$



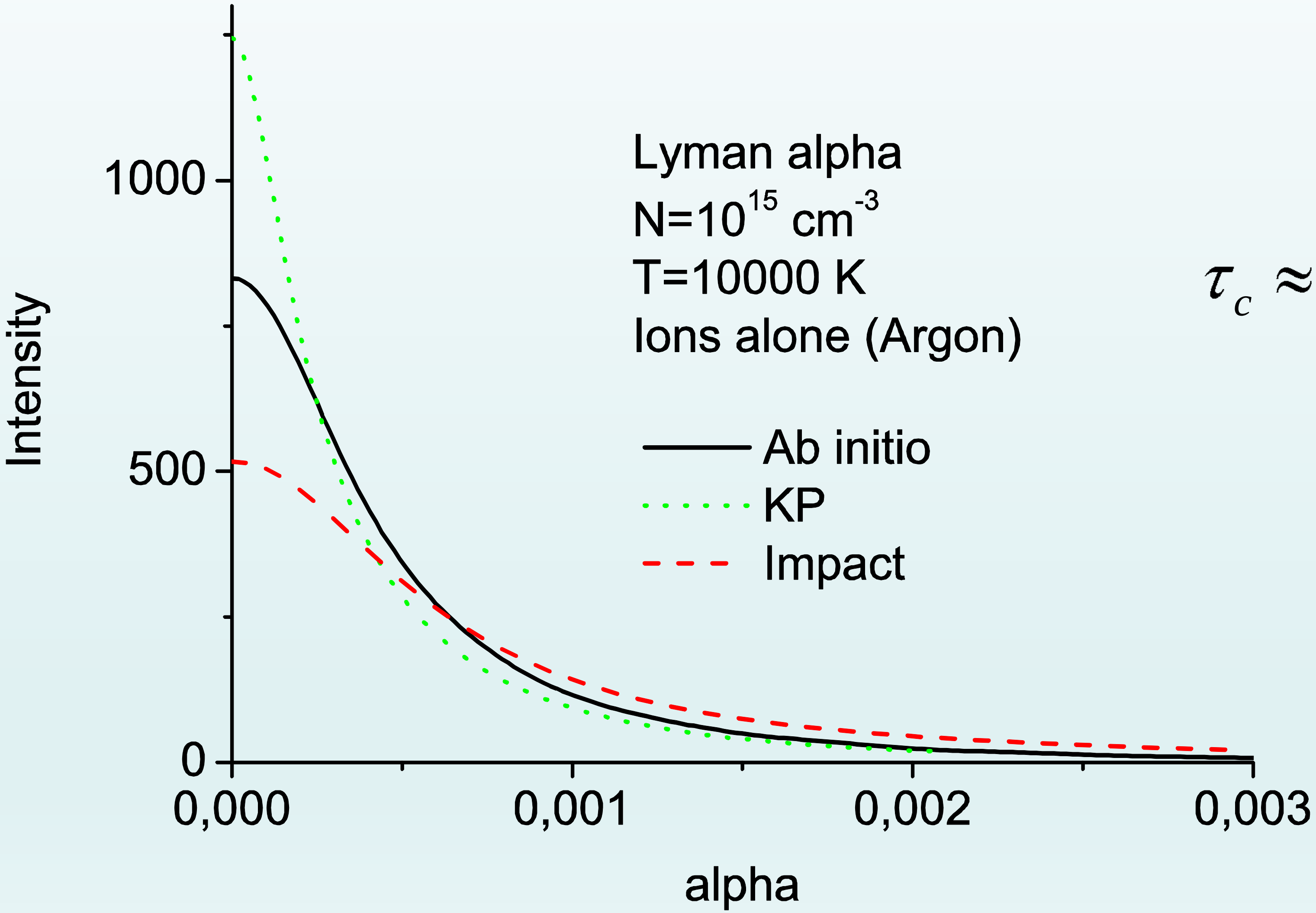
$$\alpha = \frac{\Delta\lambda}{E_0},$$

$[\Delta\lambda] \equiv$ angström

$[E_0] \equiv$ CGS, (Griem)

Lyman α with ions alone, N=10^{15} cm^{-3}

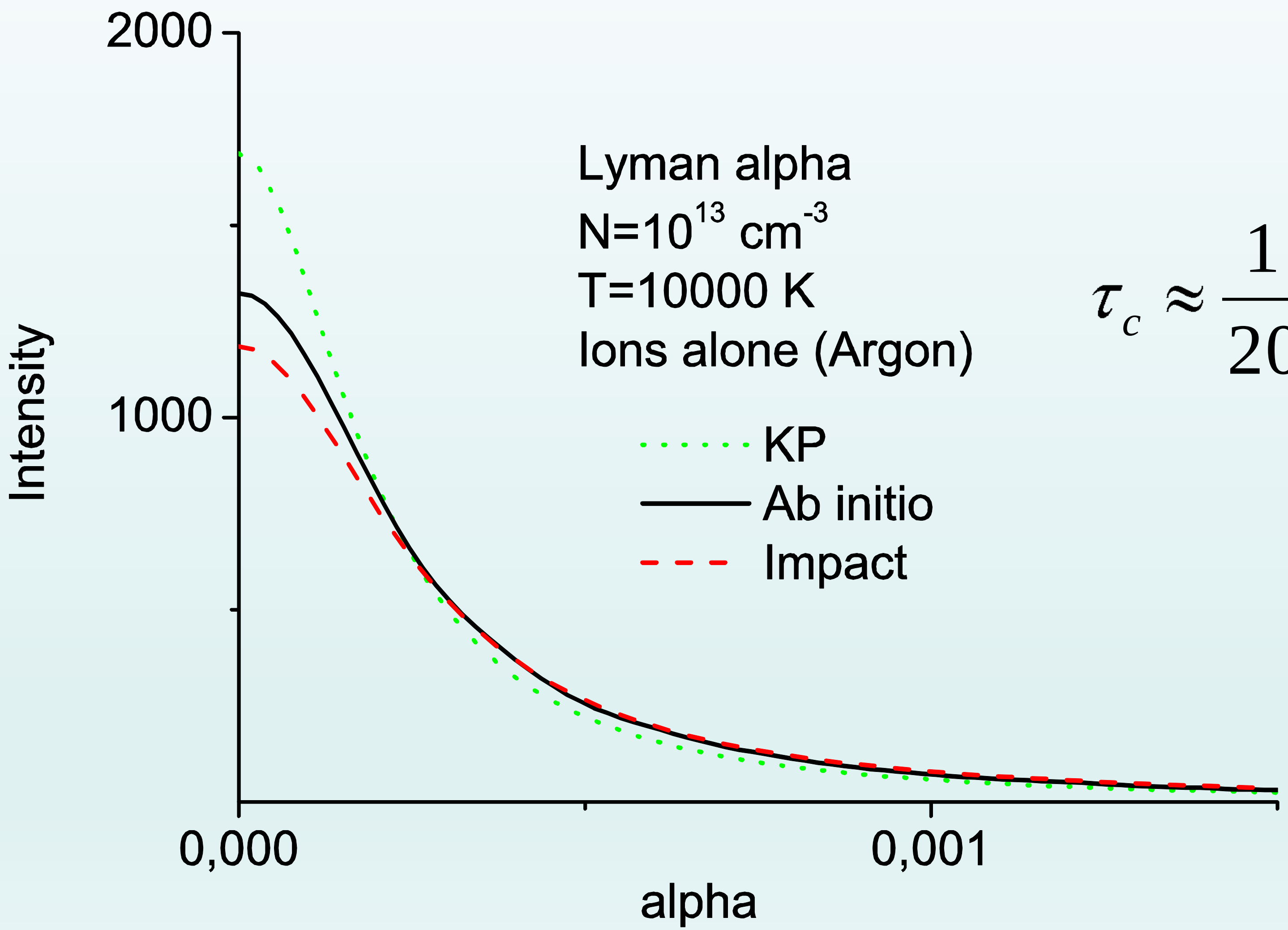
- KP profile 40 % narrower than the ab initio profile



$$\tau_c \approx \frac{1}{3}\tau_i$$

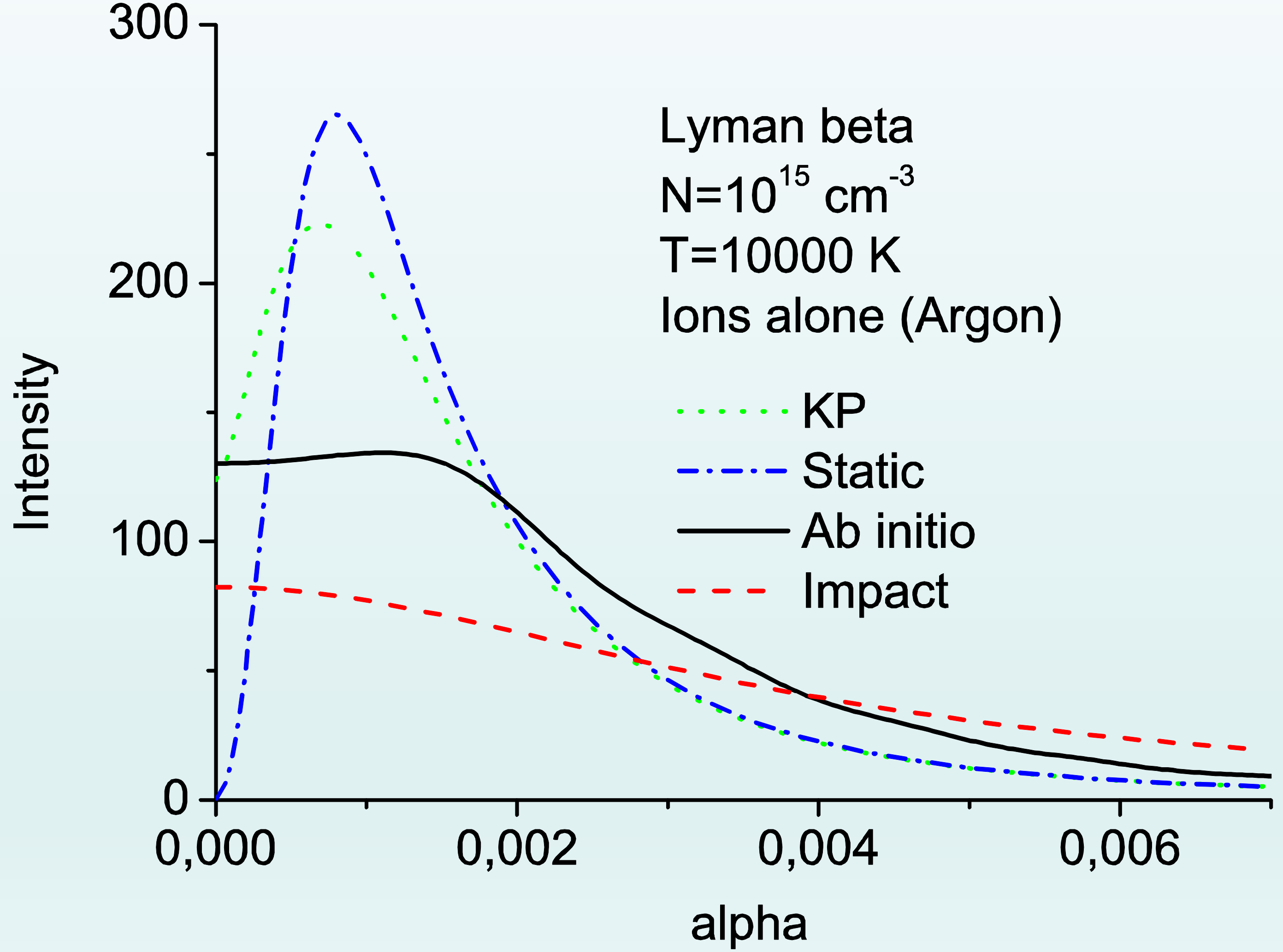
Lyman α with ions alone, N=10^{13} cm^{-3}

- Near impact conditions



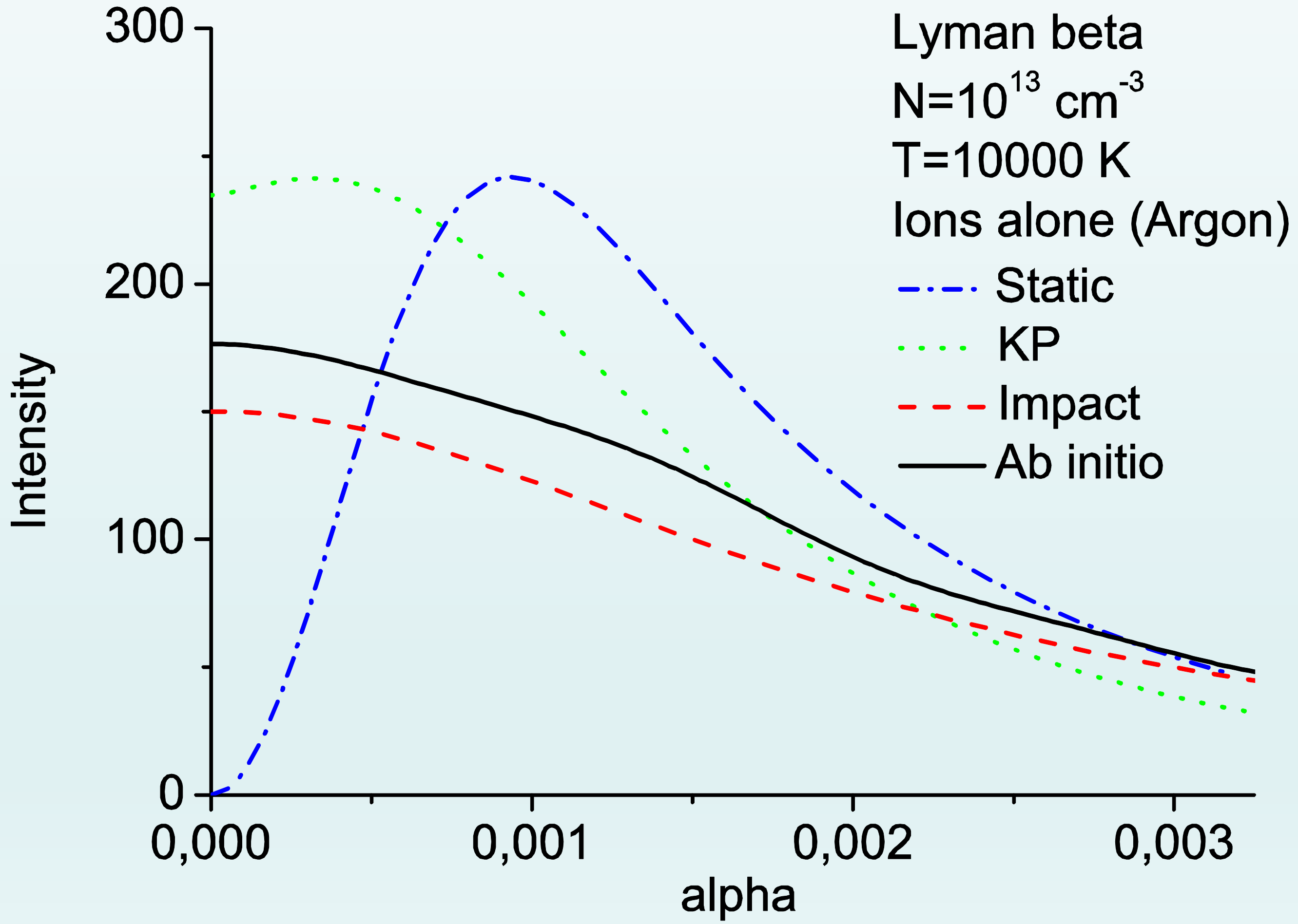
$$\tau_c \approx \frac{1}{20}\tau_i$$

Lyman β with ions alone, N=10^{15} cm^{-3}



Lyman β ions alone: near impact ($N=10^{13}$ cm^{-3})

- Near impact: 20% difference between ab initio and impact profile
- The KP remains too static



Improving the stochastic process: memory effects ?

Other stochastic process

$$v(t|E) = \frac{2}{\sqrt{\pi}} \sqrt{\nu'_E} \exp(-\nu'_E t^2)$$ Normal process

$$w(t|E) = \nu'_E t \exp(-\nu'_E t^2)$$ Weibull process with shape k=2

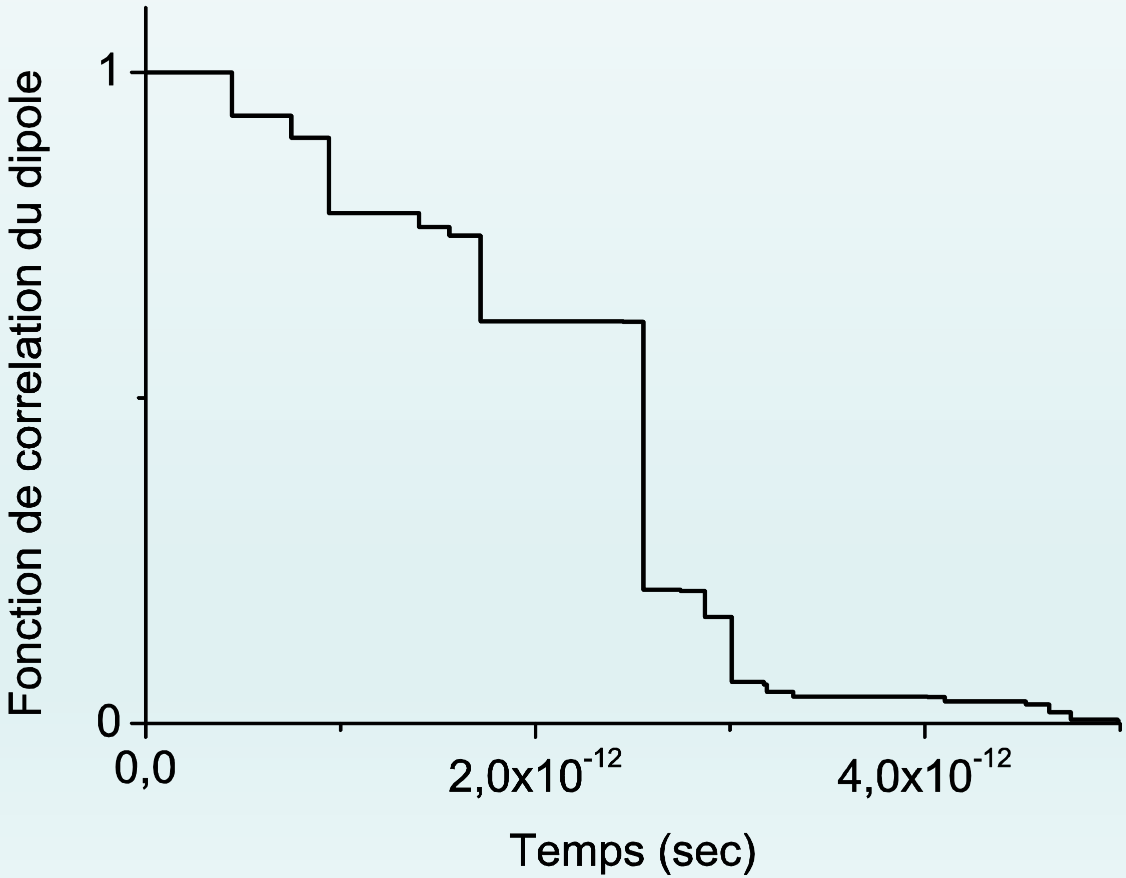
Weibull processes are used to model time-to-failure in the industry. Here the jumping frequency (failure rate) increases with time.

We use a simulation of the stochastic process

Simulation of the stochastic process

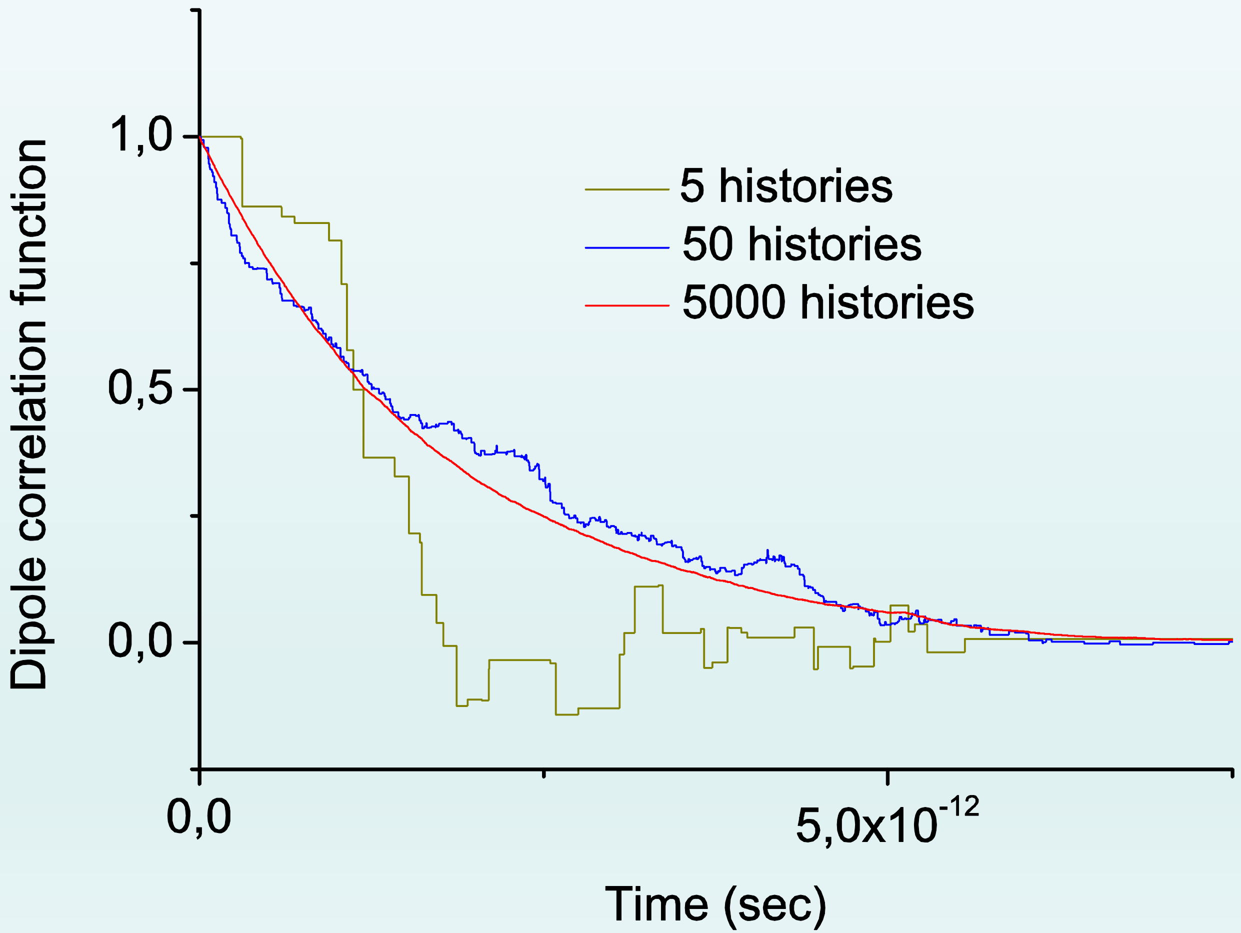
Lyman alpha dipole correlation function

Microfield are generated with the 4 PDF P(E),Q(E), v(E,t),w(E,t)



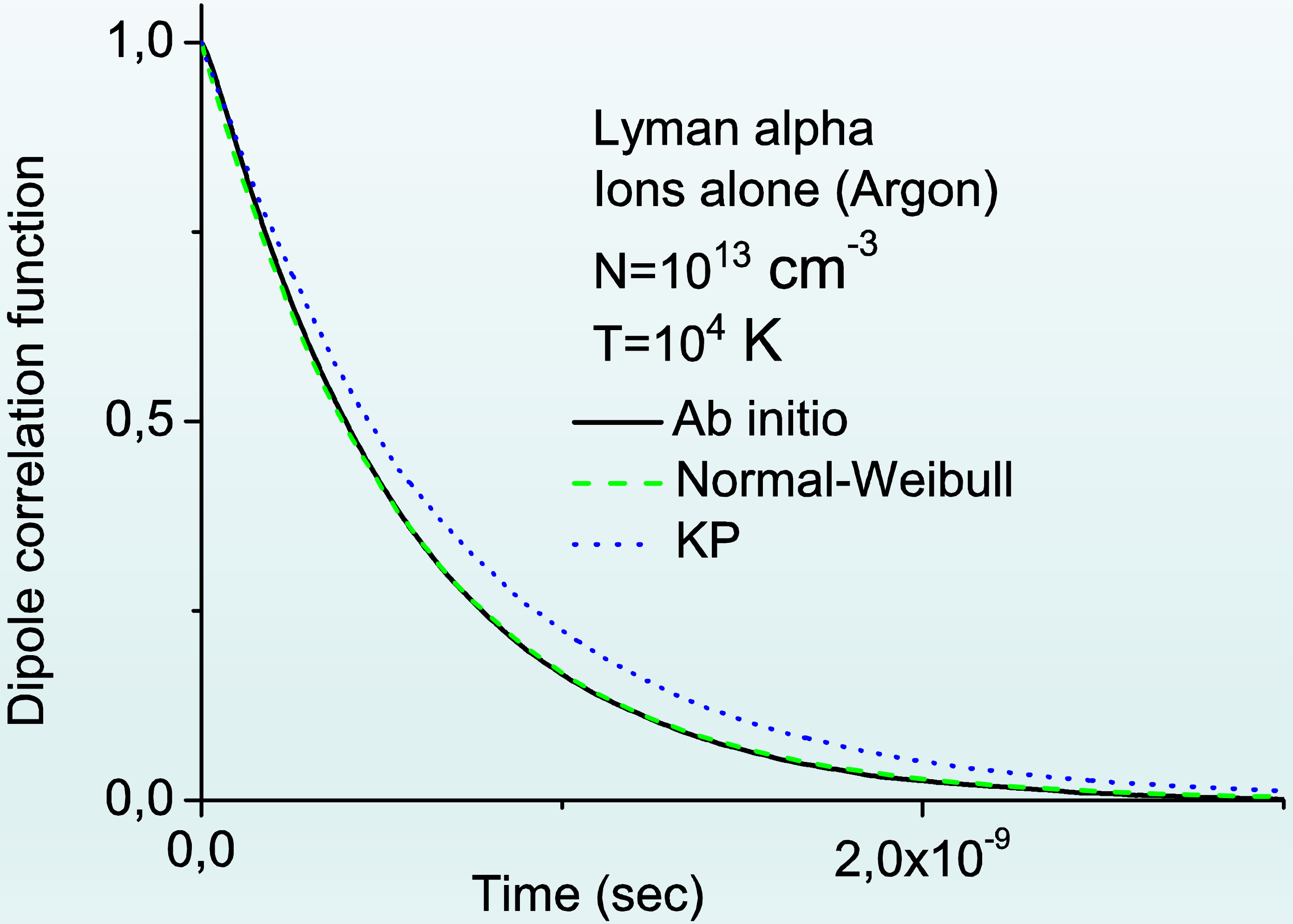
Simulation: discrete to continuum

An average over a large number of histories is required



Improving the stochastic process: memory effects ?

- Improvement for the low density (near impact) cases

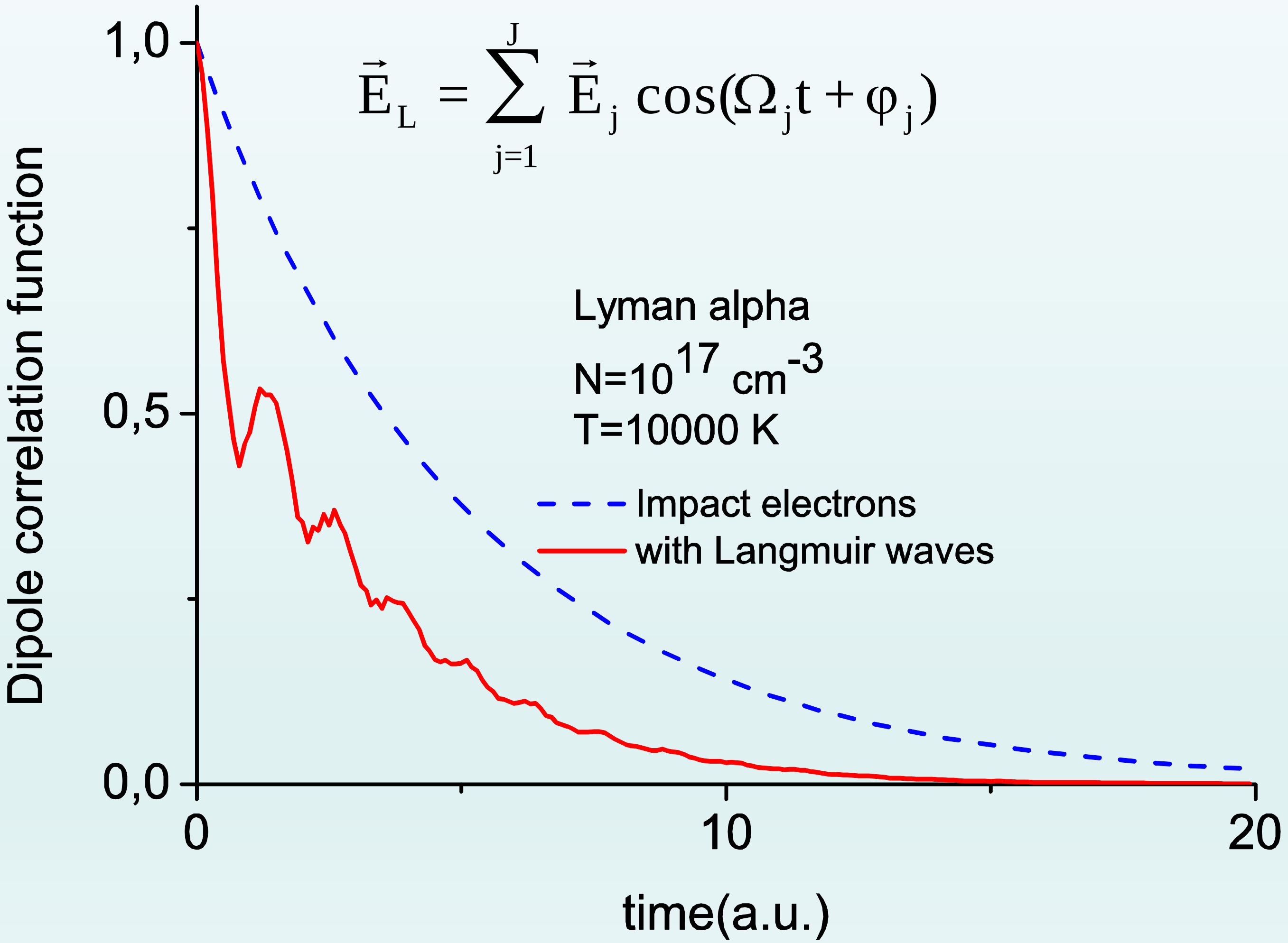


Out of equilibrium microfields : Langmuir waves

-Collective electron oscillations and waves

-Model of E. Lifshitz for the stochastic Langmuir electric field

$$\vec{E}_L = \sum_{j=1}^{J} \vec{E}_j \cos(\Omega_j t + \varphi_j)$$



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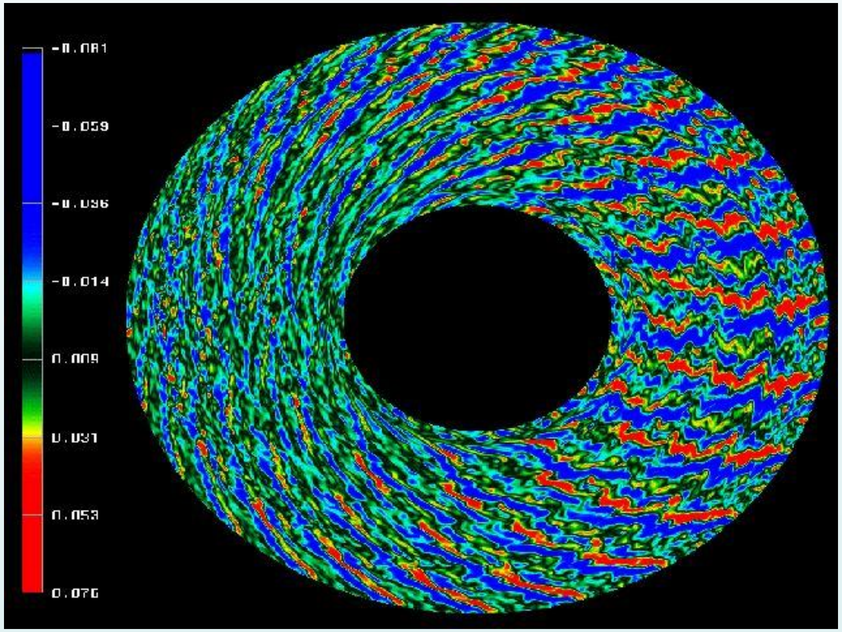
Fluctuation of plasma parameters

Non thermal fluctuations with different causes: strong gradients, hydrodynamic perturbations, drift waves,…

- Interstellar medium
- Magnetic fusion (ITER)

measures of Gamma PDF

$$p(Y) = \frac{\alpha^{\beta}}{\Gamma(\beta)} Y^{\beta-1} \exp(-\alpha Y),$$

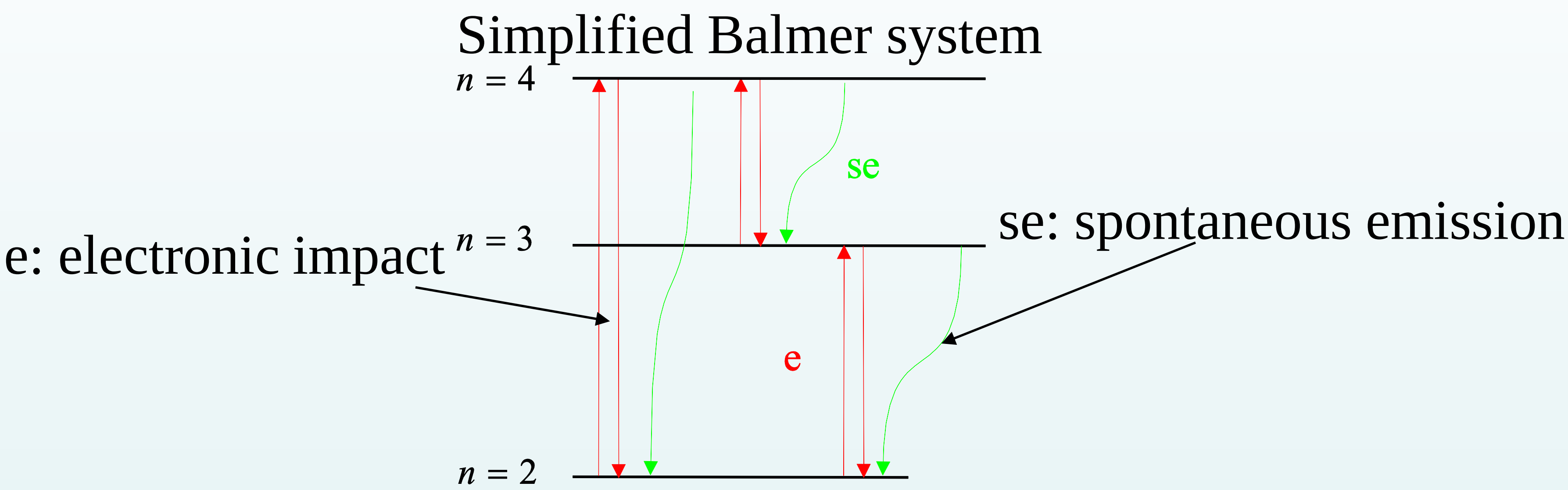


e.g. temperature fluctuation rates r=ΔT/<T>≈1 in edge plasmas

$$< T > = \frac{\beta}{\alpha}$$

$$r = \frac{1}{\sqrt{\beta}}$$

Collisionnal radiative model with fluctuations



Atomic populations X(t)

$dX(t)/dt=M(Y(t))\ X(t)$

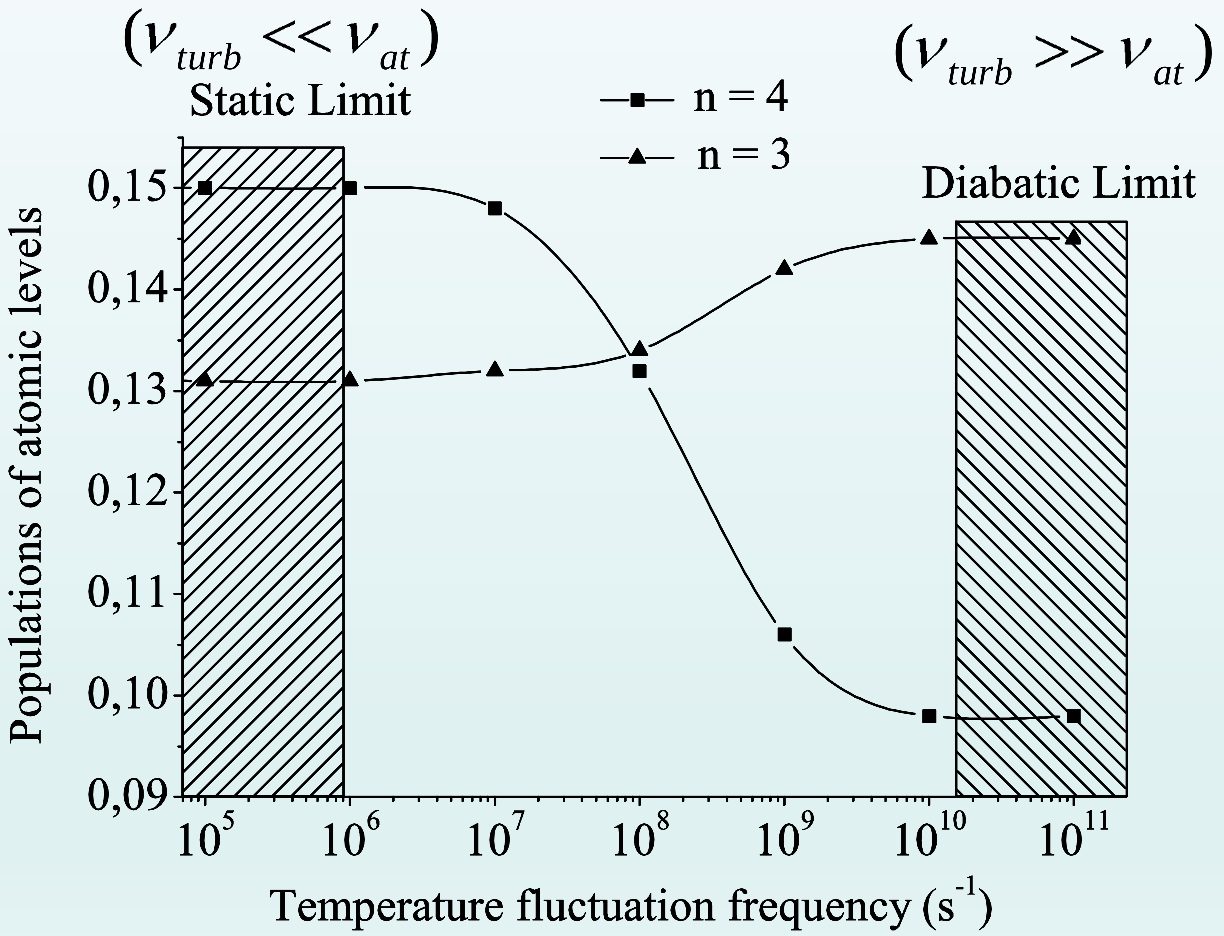
The matrix M contains the transition rates

Y(t) is the fluctuating plasma parameter (T_e or N_e)

Intensity ratio of the Balmer H_α and H_β

Temperature fluctuations with a Gamma PDF

$<T>=2$ eV, fixed density $N=10^{13}$ cm^{-3}, $\Delta T/<T>\approx 0.9$



H_α /H_β intensity ratio increases by 70 %

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- Stochastic processes allow a fast modeling of radiative processes in a plasma
- They make use of statistical properties of the plasma:

: PDF and correlation function of the fluctuating variable

- They are flexible with different possible choices for the waiting time distribution (accuracy needs to be checked by comparison to other approaches like ab initio simulations)
- Application in progress:

Collisionnal-radiative model with fluctuations

Neutral transport in a turbulent plasma

Astrophysics?

Perspectives:

Pour les profils,

- D’autres calculs sont possibles avec des processus différents
- Possibilité de traiter des effets de turbulences: champ collectif de Langmuir

Pour les cinétiques de populations atomiques,

- Prise en compte de fluctuations pour des modèles collisionnels-radiatifs de systèmes plus réalistes, avec la possibilité de comparaisons avec des résultats expérimentaux (en cours – postdoc: H et Be)

Approximation d'impact

L'approximation d'impact binaire est valide à la fois pour les électrons et ions pour les densités très faibles:

- Elargissement ionique (Hydrogène): $N_e \leq 10^{12}$ cm^{-3}, T = 1 − 100 eV

Modèle proposé par: H. Griem, A. Kolb, K. Shen (1959)

H. Baranger

Φ: opérateur de collision

Avec un développement au second ordre: $\Phi = f\left(\left\langle \vec{E}(0)\vec{E}(t)\right\rangle\right)$

Propriété statistique importante pour la forme de raie: la fonction d'autocorrélation du microchamp

$$C(t) = \exp(-\Phi t) \qquad L(\Delta\omega) = -\frac{1}{\pi}\,\mathrm{Re}\,\frac{1}{i\Delta\omega - \Phi}$$

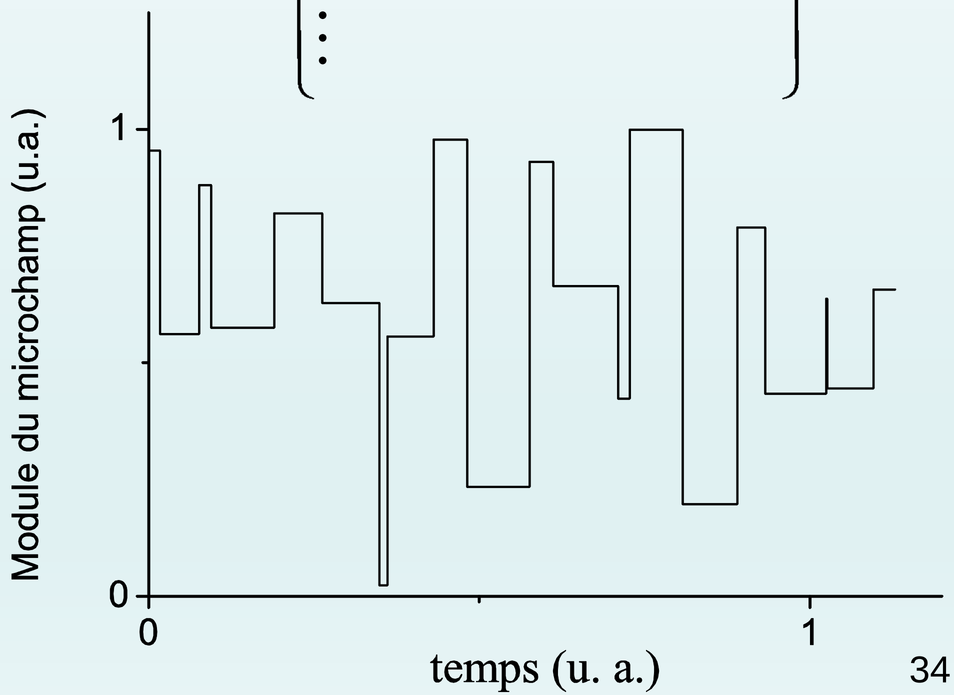
Processus de renouvellement: premier saut

Le microchamp est constant par paliers

$$\vec{E}(t) = \begin{cases} \vec{E}_1, & 0 \le t \le t_1 \\ \vec{E}_2, & t_1 \le t \le t_2 \\ \vdots & \\ \vec{E}_n, & t_{n-1} \le t \le t_n \\ \vdots & \end{cases}$$

Premier saut
- t=0 pas de sauts
- Le module du microchamp est distribué selon la PDF P(E)
- Le temps d'attente obéit à la PDF conditionnelle

$$\nu(t|E)$$



Processus de renouvellement: prochains sauts

- Le module du microchamp est distribué avec la PDF $Q(\vec{E})$
- Le temps d'attente pour tous sauf le 1er saut obéit à la PDF $w(t|\vec{E})$

Comment obtenir Q et w ? Nous avons besoin de la stationnarité du processus, et nous trouvons:

$$\begin{cases} Q(\vec{E}) = \dfrac{v(0|\vec{E})P(\vec{E})}{\langle v(0|\vec{E})\rangle_s} \\ w(t|\vec{E}) = \dfrac{-\dot{v}(t|\vec{E})}{v(t|\vec{E})} \end{cases}$$

où $<\ldots>_S$ est une moyenne sur P

Les propriétés statistiques du processus sont données par P et v

Choix du processus stochastique: Processus Kangourou

Brissaud et Frisch utilisent le processus kangourou (KP), un processus Markovien donc sans mémoire.
Pour le KP, on obtient que w = v, et

$$\mathrm{w}(\mathrm{t}|\mathrm{E}) = \mathrm{v}(\mathrm{E}) \exp(-\mathrm{v}(\mathrm{E})\, \mathrm{t})$$

Où v(E) = v(0 | E) est la fréquence de saut
Ceci conduit à la solution pour l'opérateur d'évolution avec le KP:

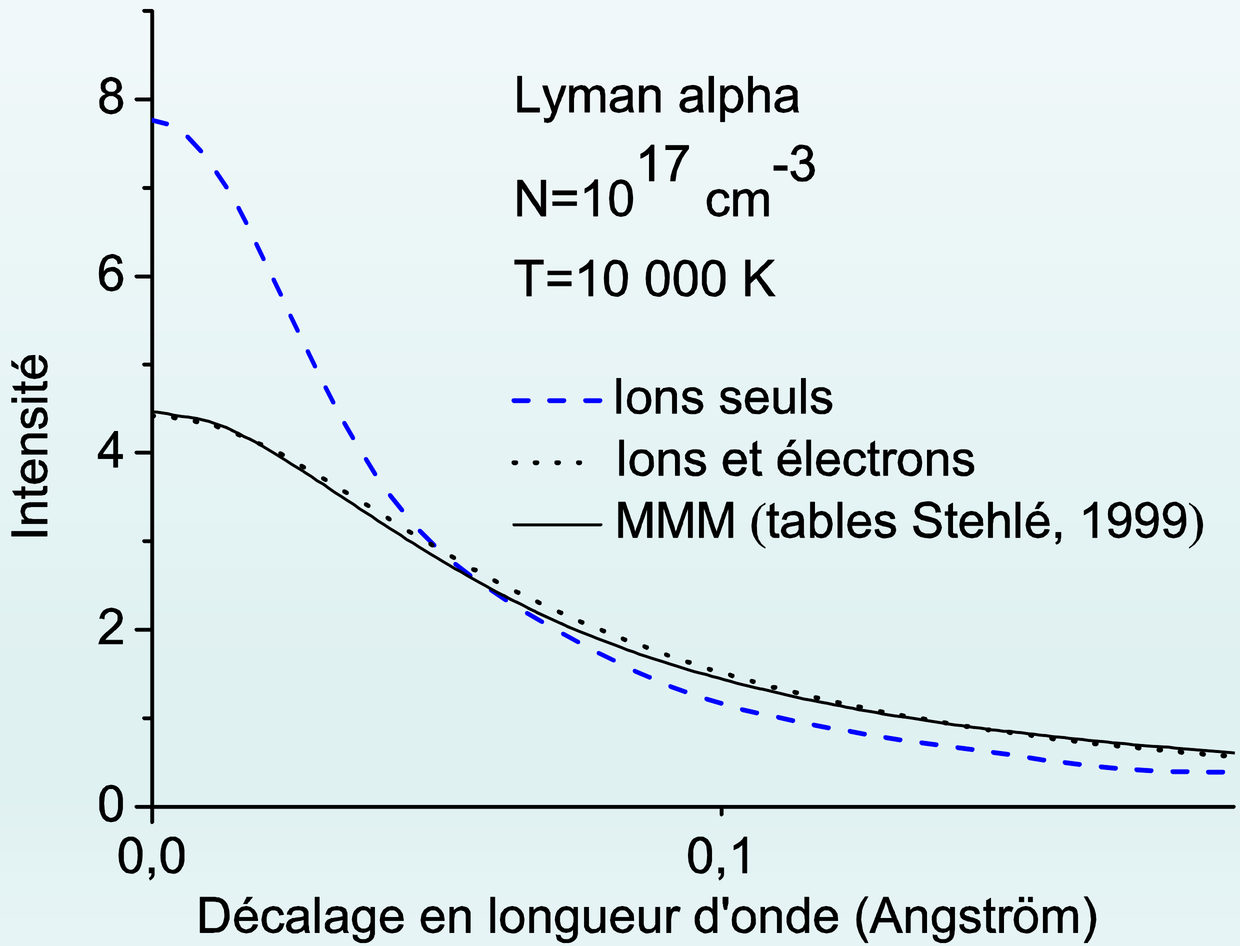
$$\left\langle \tilde{U}(\omega) \right\rangle_{KP} = \left\langle \tilde{U}(\omega'|E) \right\rangle_s + \left\langle \mathrm{v}(E) \tilde{U}(\omega'|E) \right\rangle_s$$
$$\left[\left\langle \mathrm{v}(E) \right\rangle_s - \left\langle \mathrm{v}^2(E) \tilde{U}(\omega'|E) \right\rangle_s \right]^{-1} \left\langle \mathrm{v}(E) \tilde{U}(\omega'|E) \right\rangle_s$$
$$with \quad \omega' = \omega + i\,\mathrm{v}(E)$$

Moyenne statique

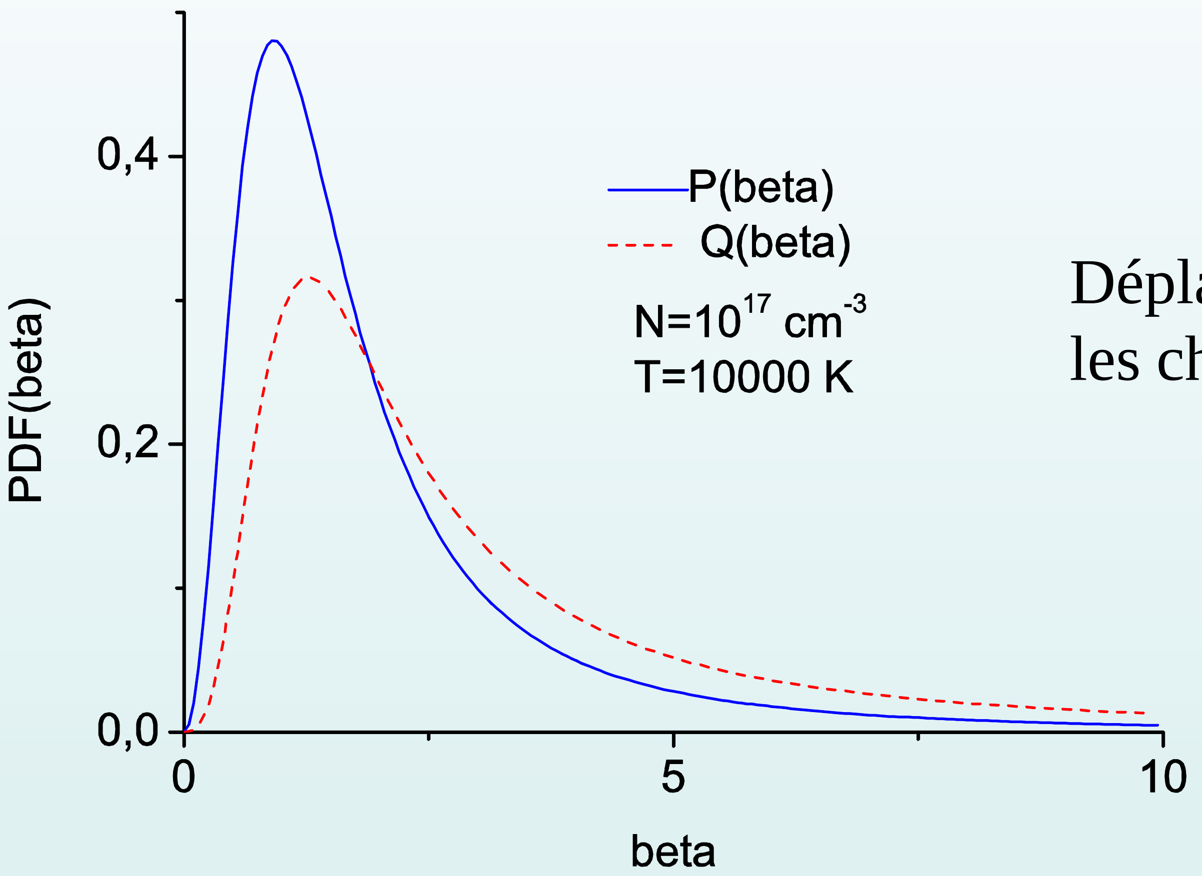
Résultats du processus kangourou pour la raie Lyman α

R. Stamm, R. Hammami et al. *Baltic Astronomy.*, 2011

R. Hammami et al. *J. Phys. Conf. Ser.*, 2012



PDFs du processus



Déplacement de Q vers les champs forts

Microchamp normalisé: $\beta = \dfrac{E}{E_0}$, $E_0 = \dfrac{e}{r_0^{\,2}}$, $r_0^{\,3} N \approx 1$

Utilisation de la corrélation du microchamp

Nous supposons le plasma isotrope, et utilisons $P(E) = P(\vec{E})4\pi E^2$

P(E) est connue à partir de la théorie cinétique (Hooper 1968)

Nous pouvons relier la fonction d'autocorrélation Γ_{PR} du processus de renouvellement à $v(t \mid E)$:

$$\Gamma_{PR}(t) = \int_0^\infty dE\, E^2 P(E) \int_t^\infty dt'\, v(t' | E)$$

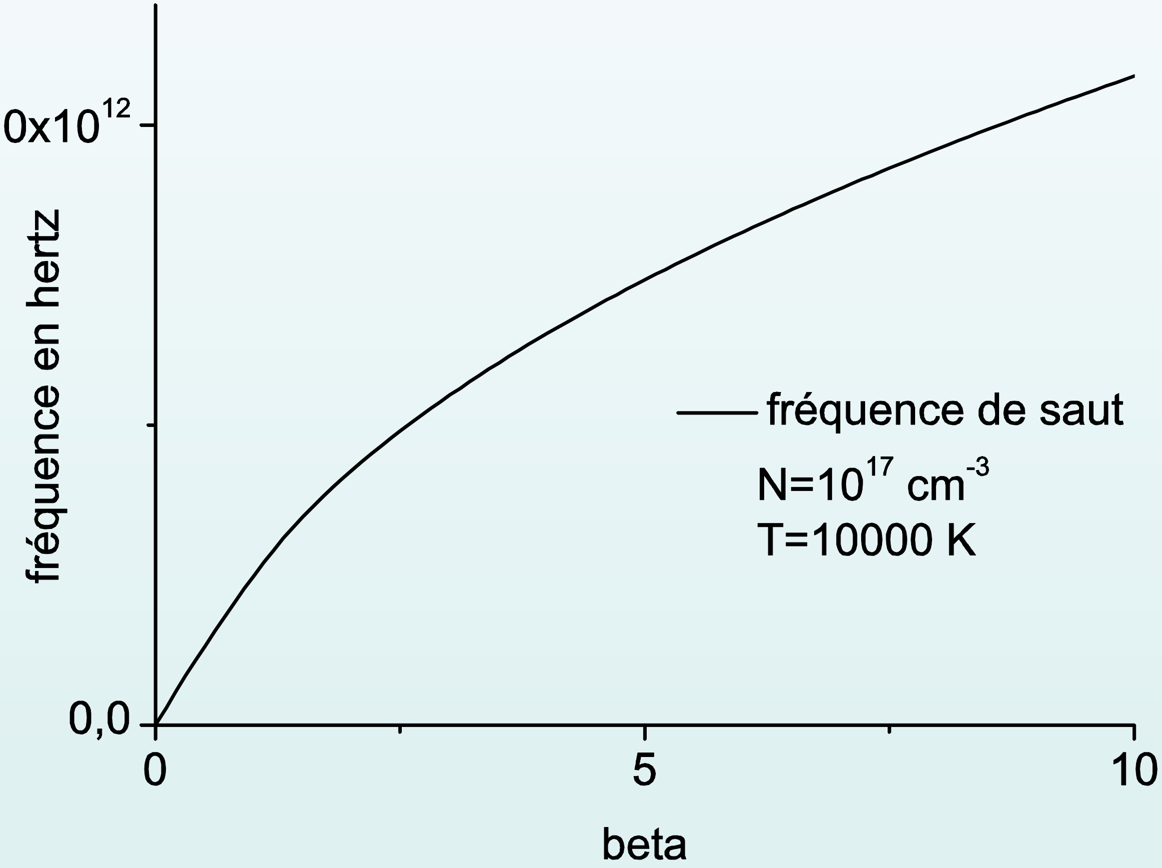
Nous pouvons imposer que Γ_{PR} soit égale à la fonction de corrélation du vrai microchamp :

$$\Gamma_{plasma} = \left\langle \vec{E}(0)\vec{E}(t) \right\rangle$$

Cette quantité a été établie par Frisch et Brissaud à partir de la théorie cinétique du plasma.

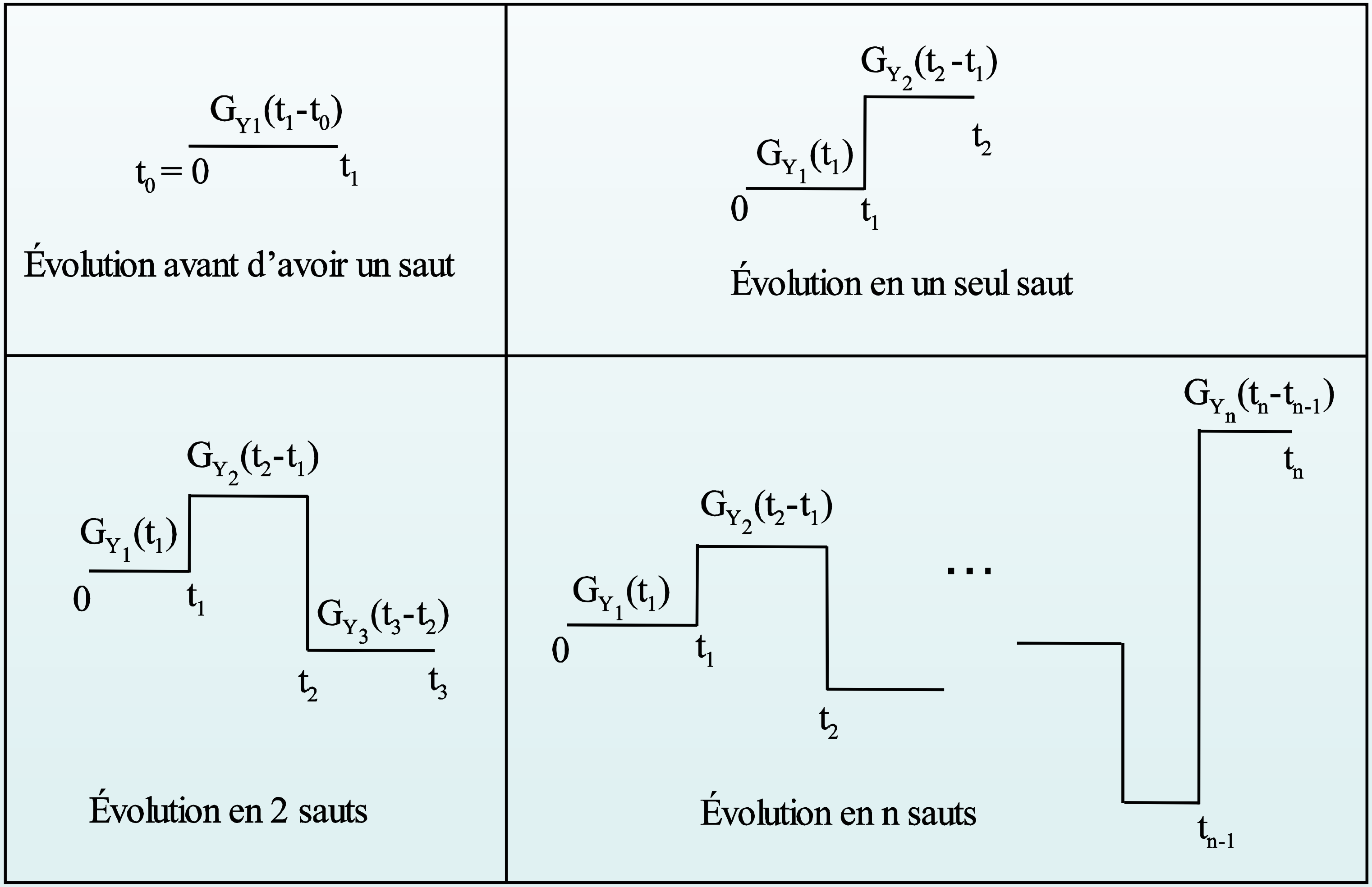
Jumping frequency

L'identification entre Γ_{RP} et Γ_{plasma} donne la fréquence de sauts



Microchamp normalisé: $$\beta = \frac{E}{E_0}$$

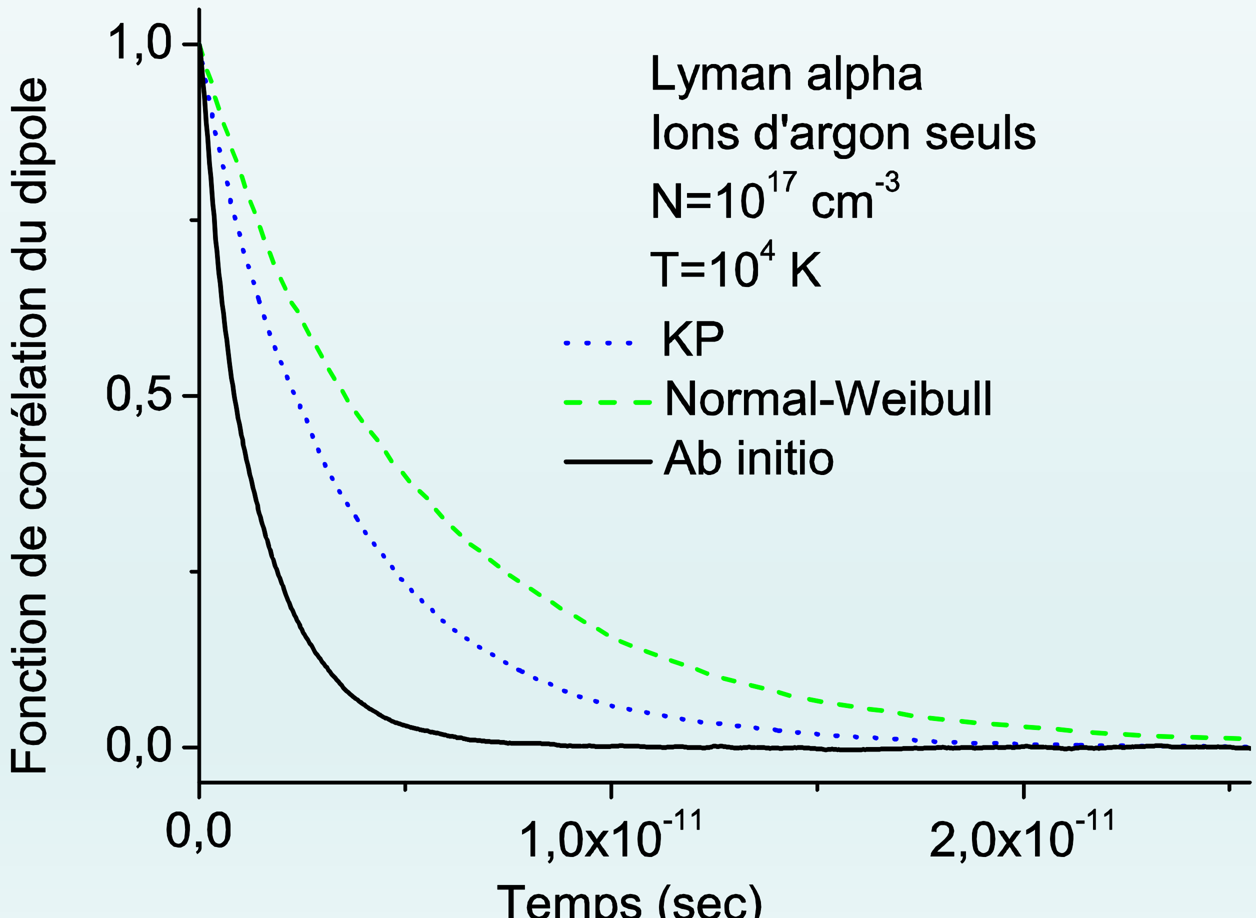
Forme de l'opérateur d'évolution:



Improving the process?

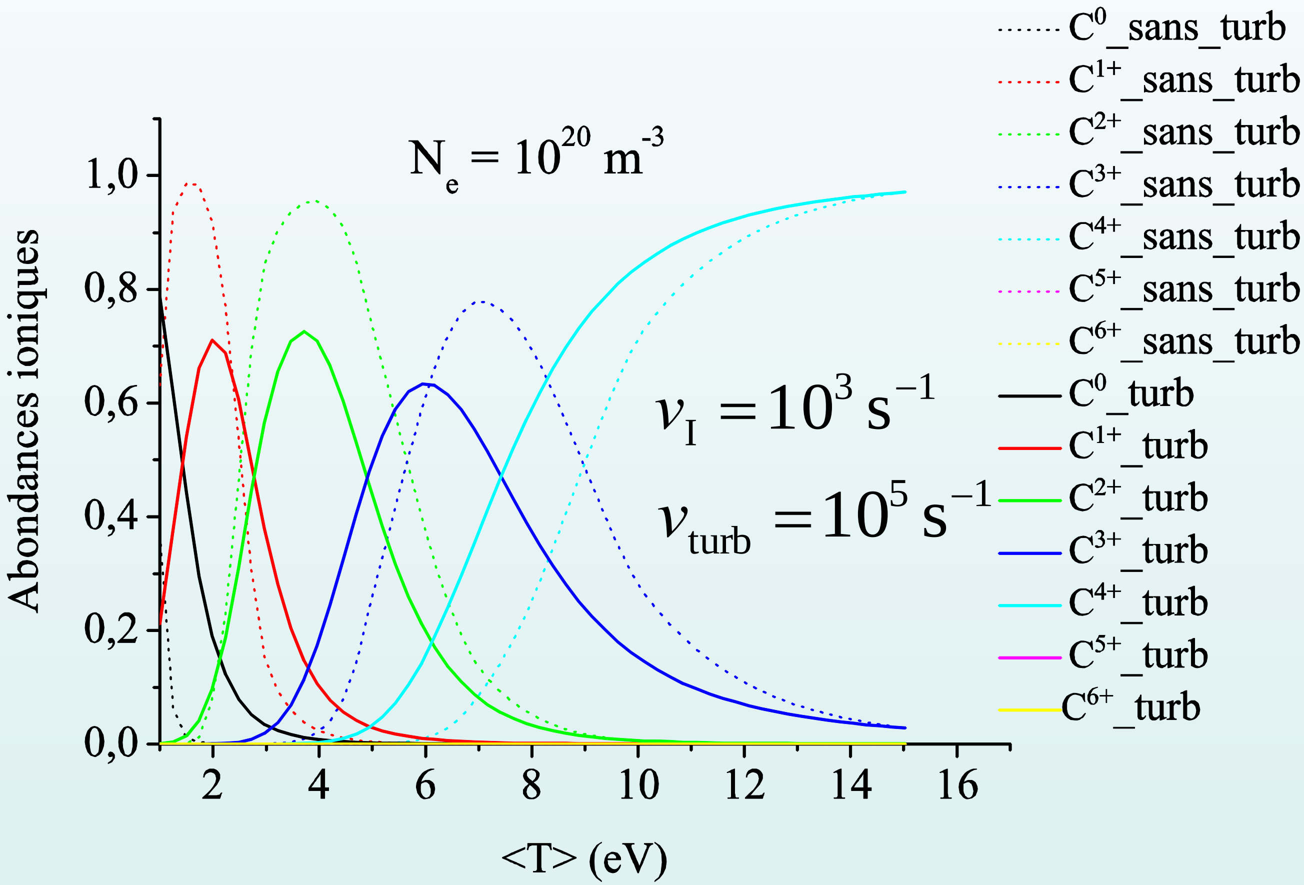
$$v(t|E) = \frac{2}{\sqrt{\pi}} \sqrt{\nu'_E} \exp\left(-\nu'_E t^2\right)$$ Processus normal (WTD Gaussienne)

$$w(t|E) = \nu'_E \, t \exp\left(-\nu'_E t^2\right)$$ Processus de Weibull



Équilibre d’ionisation du carbone: effet des fluctuations

$$\frac{d\langle N_z\rangle}{dt} = -\langle N_z N_e \alpha_{z\to z+1}\rangle - \langle N_z N_e \beta_{z\to z-1}\rangle + \langle N_{z-1} N_e \alpha_{z-1\to z}\rangle + \langle N_{z+1} N_e \beta_{z+1\to z}\rangle$$

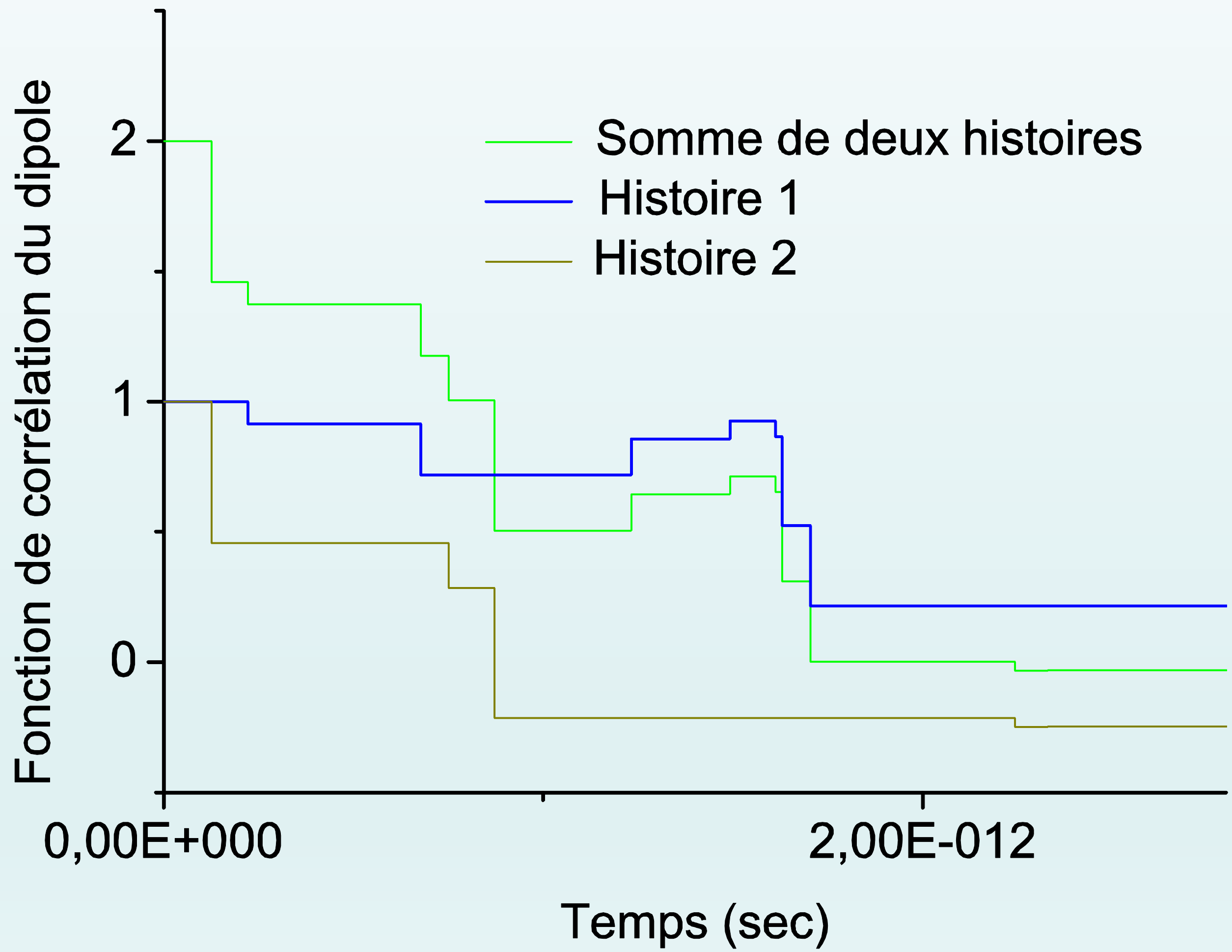


Abondances ioniques: solution avec KP

R. Hammami et al. *J. Phys. Conf. Ser.*, 2012

R. Hammami et al. *J. Nucl. Mater.*, 2013

Simulation du processus stochastique



Simulation du processus stochastique

Frerichs (1989): simulation du processus stochastique.

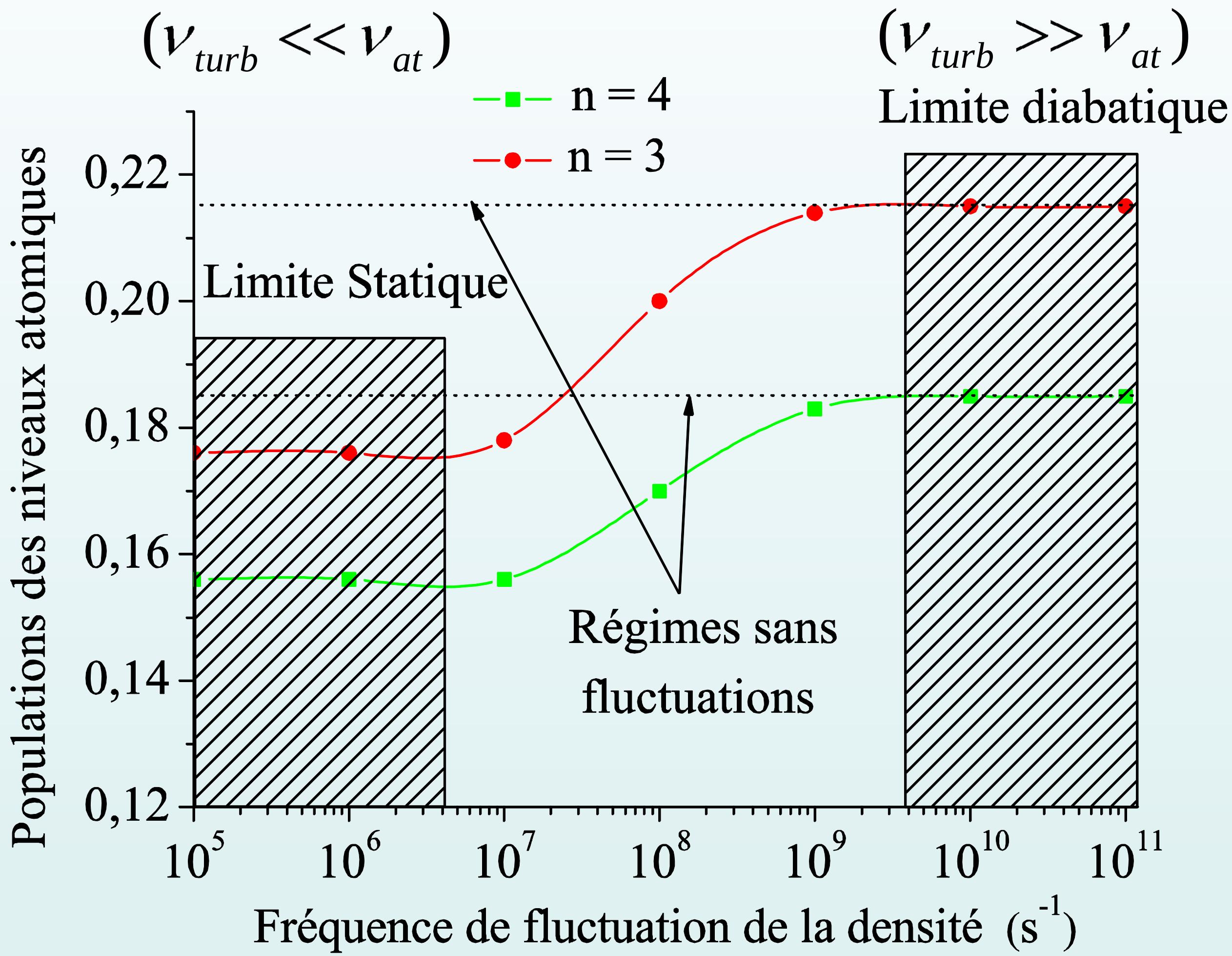
Nous générons une histoire du microchamp suivant P(E) pour le premier saut, et Q(E) pour les suivants.

Pour chaque saut, le microchamp est constant, et ainsi est l'opérateur d'évolution. L'operateur d'évolution peut être écrit comme:

$$U(t_n,0)=U(t_n, t_{n-1})\, U(t_{n-1}, t_{n-2})\ldots\ldots U(t_1,0)$$

La solution pour une histoire est un produit d'opérateurs constants

Rapports d'intensité des raies Balmer H_α et H_β



Population n=3: augmentation de 19 %

Population n=4: augmentation de 31 %

Collisionnal radiative model with fluctuations

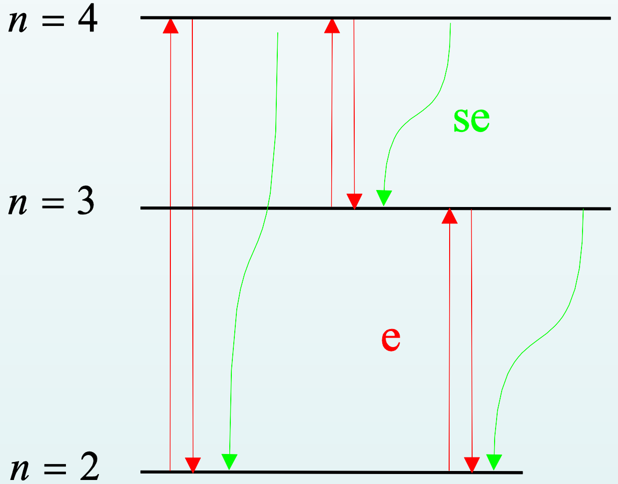
Atomic populations X(t)

dX(t)/dt=M(Y(t)) X(t)

The matrix M contains the

transition rates

Y(t) is the fluctuating plasma parameter (T_e or N_e)



Y(t) is the fluctuating plasma parameter (T_e or N_e)