ROLE OF THE COLLISIONS IN THE RADIATION REDISTRIBUTION: POLARIZATION FAR WINGS OF LINES FORMED BY SCATTERING

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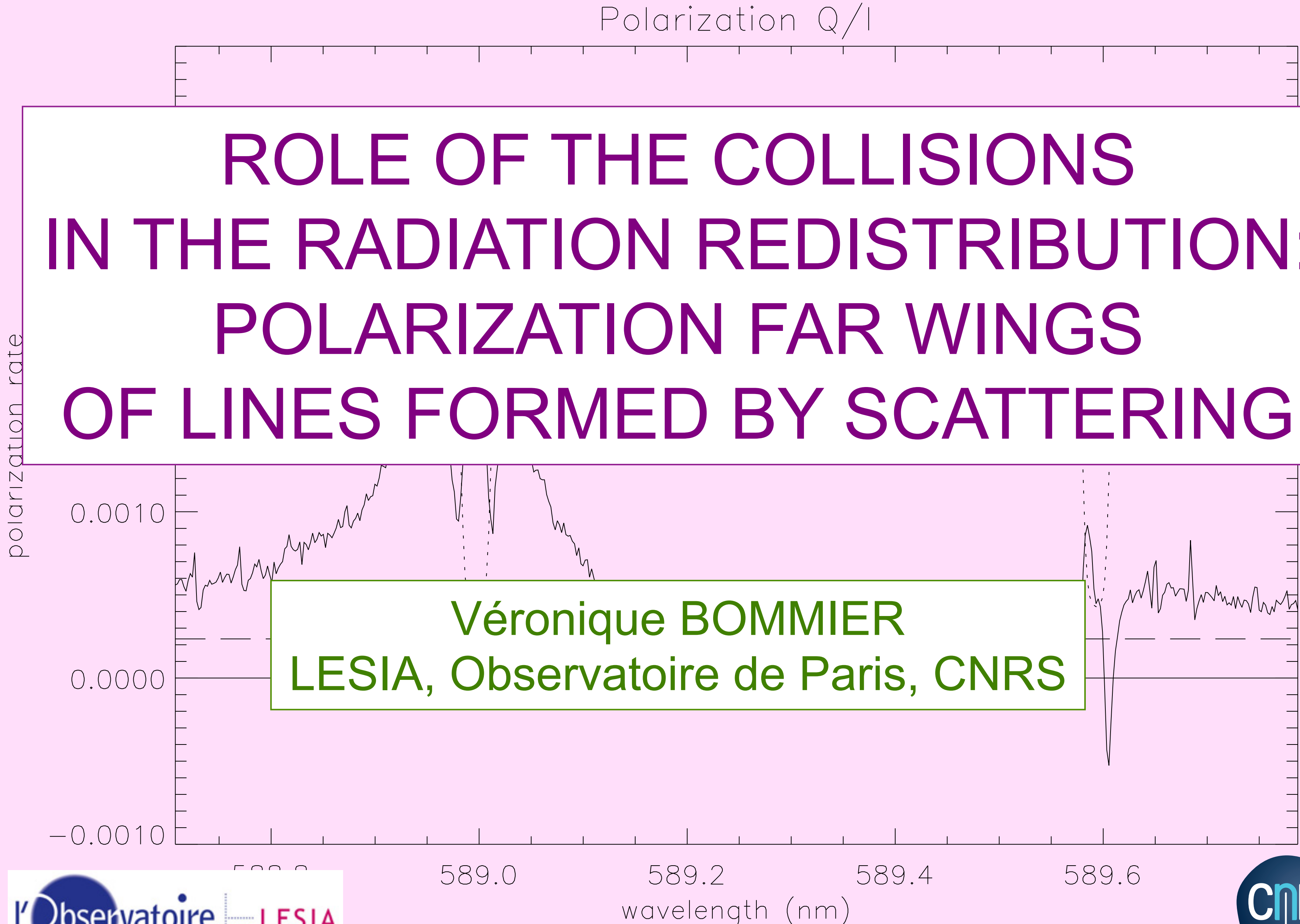


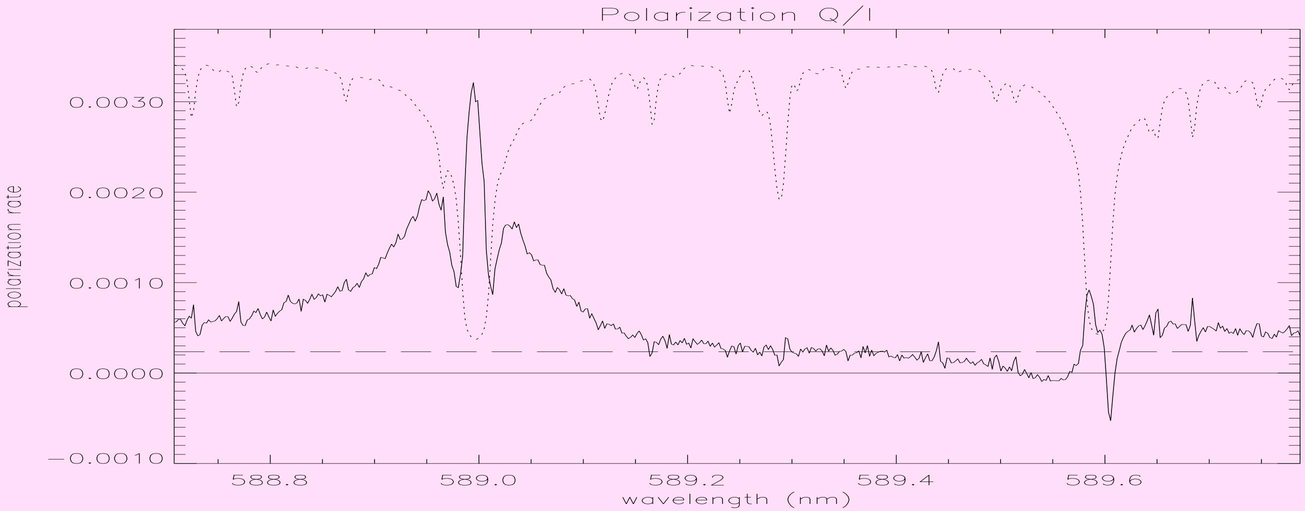
9th SCSLSA, Banja Koviljaca, 13-17 May 2013

Objective

– Modeling the line formation **including polarization**
  Magnetic field, multilevel-multiline, polarization profile, far wings
  Solving the coupled statistical equilibrium and radiative transfer equations, for the polarized atom

– Interpreting the **Second Solar Spectrum** (Stenflo, 1996)
  Linear polarization formed by scattering and observed inside the solar limb

Na I D lines



– 30% of the lines display a M-type polarization profile
  Belluzzi & Landi Degl'Innocenti, 2009, A&A 495, 577, & Belluzzi's PhD

The Markov approximation

Hamiltonian atom+radiation: $H = H_0 + V$

Schrödinger equation in interaction representation: $\mathrm{i}\hbar \dfrac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t) = \left[\tilde{V}(t), \tilde{\rho}(t)\right]$

which can be integrated in: $\tilde{\rho}(t) = \tilde{\rho}(0) + \dfrac{1}{\mathrm{i}\hbar}\int_0^t \left[\tilde{V}(t-\tau), \tilde{\rho}(t-\tau)\right]\mathrm{d}\tau$

Markov approximation: $\tilde{\rho}(t) = \tilde{\rho}(0) + \dfrac{1}{\mathrm{i}\hbar}\int_0^t \left[\tilde{V}(t-\tau), \tilde{\rho}(t)\right]\mathrm{d}\tau$

– Physical meaning: ρ does not keep memory of his past history

Validity: the characteristic ρ evolution time Γ >> the interaction correlation time τ_c
Cohen-Tannoudji (1975): the validity condition is fulfilled for weak radiation field

Consequence: the ρ finite life-time (inverse of Γ) is not taken into account in the process
the line width, or profile, is discarded from the formalism
at its place, one has

$$\int_0^{+\infty} \mathrm{e}^{-(\omega-\omega_0)\tau}\,\mathrm{d}\tau = \frac{1}{2}\delta(\omega-\omega_0) + \mathrm{i}P(\omega-\omega_0)$$

P : Cauchy Principal Value

Getting out of the Markov approximation

The Markov approximation intervenes in a perturbation development

Reporting the integral equation in the differential one

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\mathrm{i}\hbar}\left[\tilde{V}(t),\tilde{\rho}(0)\right]-\frac{1}{\hbar^2}\int_0^t\left[\tilde{V}(t)\left[\tilde{V}(t-\tau),\tilde{\rho}(t-\tau)\right]\right]\mathrm{d}\tau$$

Markov approximation closes the development:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\mathrm{i}\hbar}\left[\tilde{V}(t),\tilde{\rho}(0)\right]-\frac{1}{\hbar^2}\int_0^t\left[\tilde{V}(t)\left[\tilde{V}(t-\tau),\tilde{\rho}(t)\right]\right]\mathrm{d}\tau$$

Getting out of the Markov approximation is pursuing the perturbation development

at order-4:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\hbar^4}\int_0^t\mathrm{d}\tau\int_0^{t-\tau_1}\mathrm{d}\tau\int_0^{t-\tau_1-\tau_2}\mathrm{d}\tau\left[\tilde{V}(t),\left[\tilde{V}(t-\tau_1),\left[\tilde{V}(t-\tau_1-\tau_2),\left[\tilde{V}(t-\tau_1-\tau_2-\tau_3),\tilde{\rho}(t-\tau_1-\tau_2-\tau_3)\right]\right]\right]\right]$$

Markov approximation closes again the development

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\hbar^4}\int_0^t\mathrm{d}\tau\int_0^{t-\tau_1}\mathrm{d}\tau\int_0^{t-\tau_1-\tau_2}\mathrm{d}\tau\left[\tilde{V}(t),\left[\tilde{V}(t-\tau_1),\left[\tilde{V}(t-\tau_1-\tau_2),\left[\tilde{V}(t-\tau_1-\tau_2-\tau_3),\tilde{\rho}(t)\right]\right]\right]\right]$$

and so on.

Resummation

The statistical equilibrium equation remains the same as usual, except that in place of the δ function, at the profile place, appears a quantity of the generic form

Perturbation development manually written

$$\varphi\left\{1-\frac{A_{ba}}{2}\varphi+\frac{A_{ba}^{2}}{2^{2}}\varphi^{2}-\frac{A_{ba}^{3}}{2^{3}}\varphi^{3}+\ldots\right\}$$

One sees that it behaves as

$$\varphi\left\{\sum_{n=0}^{\infty}\left[-\frac{A_{ba}}{2}\varphi\right]^{n}\right\}$$

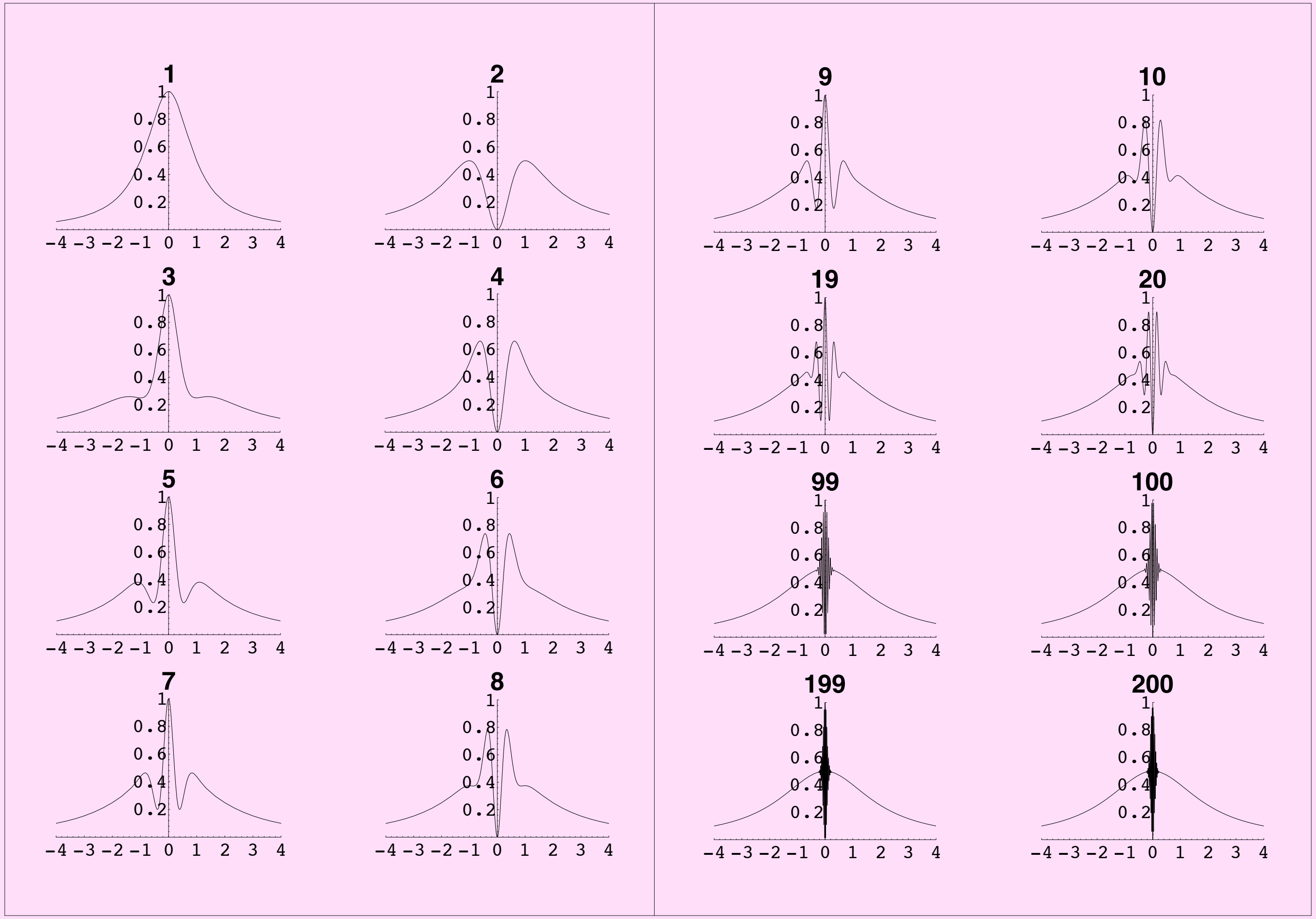
which can be resummed in

$$\frac{\varphi}{1+\frac{A_{ba}}{2}\varphi}$$

which introduces A_{ba} as a half-half-width in the profile

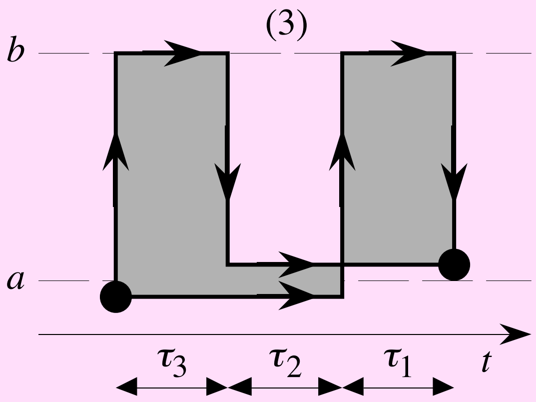
The resummed theory is non-perturbative

Visualization of the resummation effect



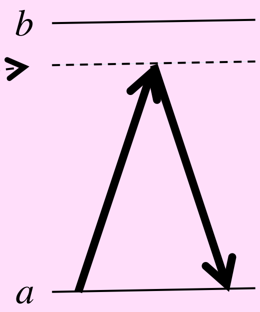
2^{nd} effect: new term at order-4 in the emissivity

New processes appear at order-4, that can be represented as:



The two transition amplitude do not stay at the same time in the upper level b

The b level is « never populated », or « virtual »
There is no absorption, nor emission
There is only scattering, with frequency conservation
This is Rayleigh scattering (can be generalized to Raman scattering)
This intervenes in the far wings



There is frequency coherence between the « absorbed » and the « emitted » photons,
Such a coherence which is rendered impossible by the Markov approximation

2nd effect: new term at order-4 in the emissivity

$$\varepsilon =$$

order-2 $\quad \dfrac{h\nu}{4\pi}\dfrac{\nu^3}{\nu_0^3} N \rho_{bb} A_{ba} \phi_{ba}(\nu_0 - \nu)$

order-4 $\quad + \dfrac{h\nu}{4\pi}\dfrac{\nu^3}{\nu_0^3} N \rho_{aa} B_{ab} \int \mathrm{d}\nu_1 J(\nu_1) \left[\dfrac{1}{2}\Phi_{ba}^*(\nu_0 - \nu)\dfrac{A_{ba}}{2}\Phi_a(\nu - \nu_1)\Phi_{ba}(\nu_0 - \nu_1)\right]$

$\Phi_{ba}(\nu_0 - \nu_1)$: complex profile of half-half-width γ_{ba}

$\Phi_a(\nu - \nu_1)$: complex profile of half-half-width the lower level a life-time

infinitely sharp lower level a:

$$\left[\frac{1}{2}\Phi_{ba}^*(\nu_0 - \nu)\frac{A_{ba}}{2}\Phi_a(\nu - \nu_1)\Phi_{ba}(\nu_0 - \nu_1)\right] = \frac{A_{ba}}{2\gamma_{ba}}\left\{\delta(\nu - \nu_1)\phi_{ba}(\nu_0 - \nu_1) - \phi_{ba}(\nu_0 - \nu)\phi_{ba}(\nu_0 - \nu_1)\right\}$$

The order-4 term in the emissivity:

– its integral over one or the other of the frequencies is zero
– it redistributes the frequencies inside the emission profile
– the result is a decoupling between atom and radiation

2-level atom: Redistribution Function

Analytical solution of the statistical equilibrium reported in the emissivity

Γ_R : radiative inverse life-time

Γ_I : inelastic collisions ($b \leftrightarrow a$) inverse life-time

Γ_E : elastic collisions (in b) inverse life-time

$$\gamma_{ba} = \frac{1}{2}\left(\Gamma_R + \Gamma_I + \Gamma_E\right)$$

$$\varepsilon = \frac{h\nu}{4\pi}\frac{\nu^3}{\nu_0^3} N \rho_{aa} B_{ab} \int \mathrm{d}\nu_1 J(\nu_1) \left\{ \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E}\delta(\nu - \nu_1)\phi_{ba}(\nu_0 - \nu_1) + \frac{\Gamma_R}{\Gamma_R + \Gamma_I}\frac{\Gamma_E}{\Gamma_R + \Gamma_I + \Gamma_E}\phi_{ba}(\nu_0 - \nu)\phi_{ba}(\nu_0 - \nu_1) \right\}$$

with polarization:

$$\varepsilon = \frac{h\nu}{4\pi}\frac{\nu^3}{\nu_0^3} N \rho_{aa} B_{ab} \int \mathrm{d}\nu_1 \oint \frac{\mathrm{d}\vec{\Omega}_1}{4\pi} \sum_K \left[w_{J'J}^{(K)}\right]^2 \sum_{j=0}^{3} \left[P_R^{(K)}\left(\vec{\Omega}, \vec{\Omega}_1\right)\right]_{ij} S_j\left(\nu_1, \vec{\Omega}_1\right) \quad \text{Rayleigh phase matrix}$$

$$\left\{ \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E}\delta(\nu - \nu_1)\phi_{ba}(\nu_0 - \nu_1) + \frac{\Gamma_R}{\Gamma_R + \Gamma_I + D^{(K)}}\frac{\Gamma_E - D^{(K)}}{\Gamma_R + \Gamma_I + \Gamma_E}\phi_{ba}(\nu_0 - \nu)\phi_{ba}(\nu_0 - \nu_1) \right\}$$

The collision rates weight the contributions of the different redistribution types
(coherent or complete)

Bibliography

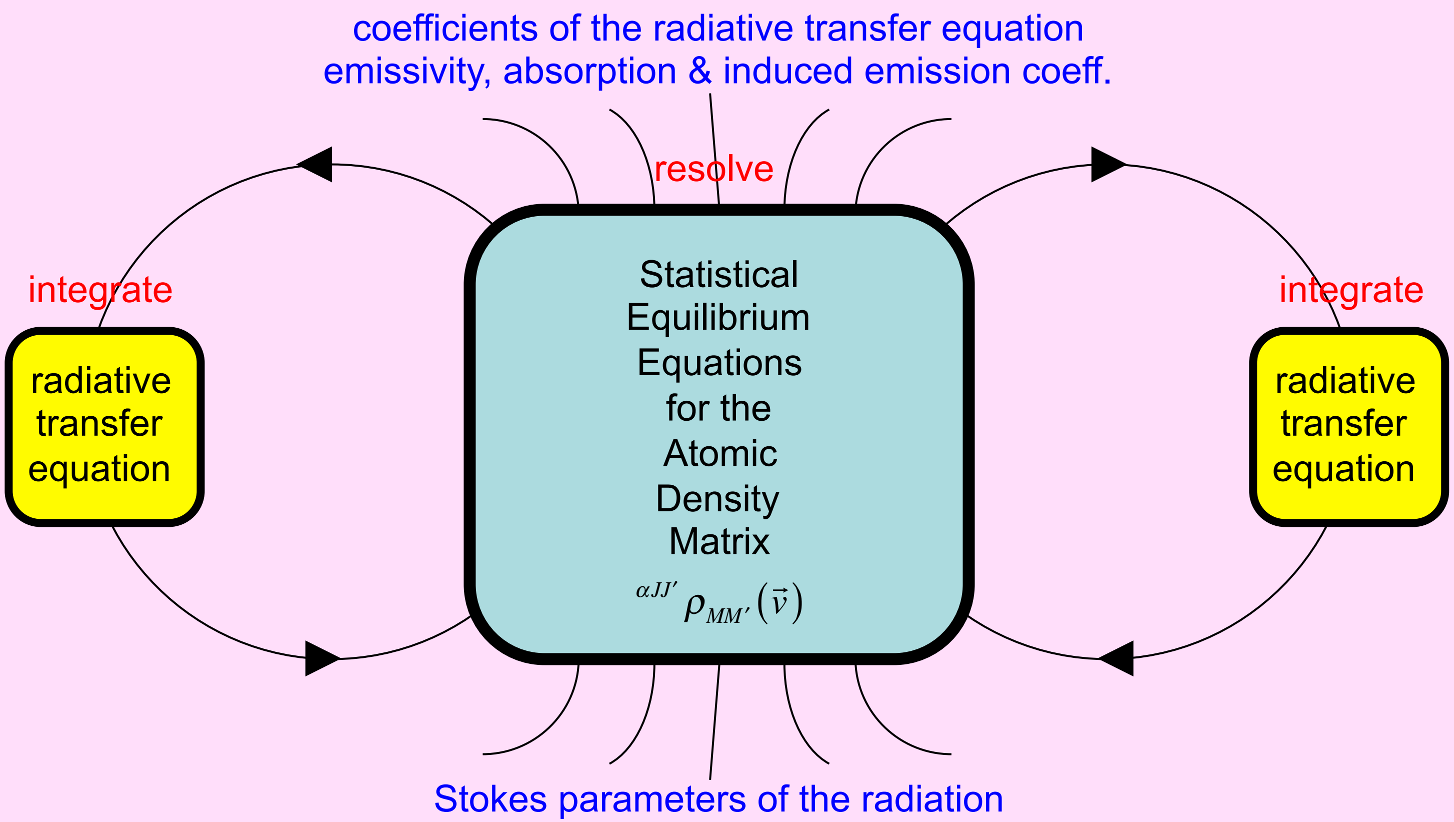
– this theory, for a 2-level atom, with polarization and magnetic field
Bommier, V., 1997, A&A, 328, 706 & 726
+ Bommier, V., 1999, ASSL 243 (SPW2), 43
for Raman scattering and Doppler redistribution
(the statistical equilibrium has to be solved for each velocity class of the atoms)

– full agreement about the redistribution functions
and the physical description of the Rayleigh scattering
with Omont, Smith, Cooper, 1972, ApJ, 175, 185

– previous papers make use of the emissivity developed in two terms,
but from empirical derivation
Hubeny, Oxenius, Simonneau, 1983, JQSRT, 29, 495
Hubeny, I., 1985, Bull. Astron. Inst. Czechosl., 36, 1
Hubeny & Lites, 1995, ApJ, 455, 376
Uitenbroek, H., 1989, A&A, 213, 360

XTAT, a code based on this theory for modeling the polarized line formation

Centered on statistical equilibrium resolution for the multilevel atom (iterative method)



Particularities

– No redistribution functions
replaced by: the 2nd term of the emissivity stemmed from the order-4 of the development
– Source function: temporarily introduced, for integrating the radiative transfer equation
– Purely numerical solution: no formal solution introduced
– thanks to Ibgui et al., 2013, A&A, 549, A126 for a new cubic short characteristics method

XTAT baby first steps (4 months old at all):