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Calculation of the Multiplet Factor ($l^n_1\, l^m_2 l^p_3$) in L-S coupling

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Abstract. Sahal-Bréchot studied the Stark broadening of isolated lines in the impact and quasistatic approximation; semi-classical formula were provided, including both dipole and quadrupole in the expansion of the electrostatic interaction between the optical electron and the perturber. Moreover the quadrupole potentiel is predominant for a certain number of complex ions. Therefore the angular factors of the quadrupole term appearing in the semi-classical expression of the width of line broadened by electron or ion is calculated in $L-S$ coupling for complex atoms, using the Fano-Racah algebra. The purpose of this paper is to provide new multiplet factor formula for more complicated configuration as $(l_1^n(L_nS_n)l_2^m(L_mS_m)l_3^p(L_pS_p))$.

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For the quadrupole interaction, the non-spherical symmetry of the r_p^{-3} potentiel is such that $J_iM_i \longrightarrow J_iM_i^{'}$ (or $J_fM_f \longrightarrow J_fM_f^{'}$) transition can occure and an angular average has to be performed. The scope of this paper is to study a new multiplet factor formula in order to perform the angular average for more complicated configuration $l_1^n l_2^m l_3^p$.

First, we construct the antisymmetric wave function of the configuration $l_1^n l_2^m l_3^p$ and adapt their reasoning for the diagonal element (ED); one obtains :

$$ED = \langle \Psi^*_{anti}(l_1^n\ l_2^m\ l_3^p\ )LS \,|\, \sum_{i=1}^{n+m+p} r_i^2 C^{(2)} \,|\, \Psi_{anti}(l_1^n\ \ l_2^m\ \ l_3^p\ )\ LS\rangle$$

- Coefficient of fractionnal parentage appears and noting that the operator $r^2C^{(2)}$ does not act on spin variable and on the electrons of the cores.

- Using the formula's (15.26) and (15.27) derived by de Shalit and Talmi (1963).

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If we couple : $\overrightarrow{L_n} + \overrightarrow{L_m} = \overrightarrow{L'}$. The reduced matrix element $(L'(L_{p-1}l_3)L_pL \| r^2C^{(2)} \| L'(L_{p-1}l_3)L_pL)$ can be written as :

$$ED = R_{mult1}(l_1 \| r^2C^{(2)} \| l_1) + R_{mult2}(l_2 \| r^2C^{(2)} \| l_2) + R_{mult3}(l_3 \| r^2C^{(2)} \| l_3)$$

With :

$$R_{mult1} = (-1)^{l_1+L+L_m+L_p}(2L+1)(2L_n+1)(2L'+1)W(L_nL_nL'L'2L_m)$$

$$W(L'L'LL2L_p)\sum_{\beta_{n-1}} n(-1)^{L_{n-1}}\left[l_1^{n-1}(\beta_{n-1})l_1)l_1^nL_nS_n\right]^2 W(l_1l_1L_nL_n2L_{n-1})$$

$$R_{mult2} = (-1)^{l_2+L+L_n+L_p}(2L+1)(2L_m+1)(2L'+1)W(L_mL_mL'L'2L_n)$$

$$W(L'L'LL2L_p)\sum_{\beta_{m-1}} m(-1)^{L_{m-1}}\left[l_2^{m-1}(\beta_{m-1})l_2)l_2^mL_mS_m\right]^2 W(l_2l_2L_mL_m2L_{m-1})$$

$$R_{mult3} = (-1)^{l_3+L+L'}(2L+1)(2L_p+1)W(L_pL_pLL2L')\times$$

$$\sum_{\beta_{p-1}} p(-1)^{L_{p-1}}\left[l_3^{p-1}(\beta_{p-1})l_3)l_3^pL_pS_p\right]^2 W(l_3l_3L_pL_p2L_{p-1})$$

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When coupling L_m and L_p, we can write $\overrightarrow{L_m} + \overrightarrow{L_p} = \overrightarrow{L''}$. In this case, one obtains

$$R_{mult1} = (-1)^{l_1+L+L''}(2L+1)(2L_n+1)W(L_nL_nLL2L'')\times$$

$$\sum_{\beta_{n-1}} n(-1)^{L_{n-1}}\left[l_1^{n-1}(\beta_{n-1})l_1)l_1^n L_n S_n\right]^2 W(l_1l_1L_nL_n2L_{n-1})$$

$$R_{mult2} = (-1)^{l_2+L+L_n+L_p}(2L+1)(2L_m+1)(2L''+1)W(L''L''LL2L_n)$$

$$W(L_mL_mL''L''2L_p)\sum_{\beta_{m-1}} m(-1)^{L_{m-1}}\left[l_2^{m-1}(\beta_{m-1})l_2)l_2^m L_m S_m\right]^2 W(l_2l_2L_mL_m2L_{m-1})$$

$$R_{mult3} = (-1)^{l_3+L+L_n+L_m}(2L+1)(2L_p+1)(2L''+1)W(L''L''LL2L_n)$$

$$W(L_pL_pL''L''2L_m)\sum_{\beta_{p-1}} p(-1)^{L_{p-1}}\left[l_3^{p-1}(\beta_{p-1})l_3)l_3^p L_p S_p\right]^2 W(l_3l_3L_pL_p2L_{p-1})$$

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SUM RULE

$$\sum_{L''L'''}(2L''+1)(2L'''+1)\begin{pmatrix} L_n & L_m & L' \\ L_p & L & L'' \end{pmatrix}\begin{pmatrix} L''' & L'' & 2 \\ L & L & L_n \end{pmatrix}\begin{pmatrix} L_n & L_m & L' \\ L_p & L & L''' \end{pmatrix}\begin{pmatrix} L_m & L_m & 2 \\ L'' & L''' & L_p \end{pmatrix} =$$

$$\begin{pmatrix} L' & L' & 2 \\ L & L & L_p \end{pmatrix}\begin{pmatrix} L_m & L_m & 2 \\ L'' & L'' & L_n \end{pmatrix}$$

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