

## CHARACTERISTIC LINE PROFILE PARAMETERS OF HYDROGEN BALMER LINES IN AN EXTERNAL ELECTRIC FIELD

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### 1. INTRODUCTION

Several recently reported measurements of the electric field strength distribution in the cathode fall region of a glow discharge (Barbeau and Jolly, 1991; Ganguly and Garscadden, 1991; Donkó et al., 1994; Videnović et al., 1996; Kuraica et al., 1997) have employed the polarization-dependent Stark splitting coupled with Doppler broadening of hydrogen Balmer lines. In most of these studies, to determine local electric field strength the thorough theory of linear Stark effect is used. However, for some purposes the knowledge of line profile parameters only, e.g. the wavelength distance  $\Delta\lambda_{pp}$  between most distinguished maximums (Kuraica et al., 1997) or the profile halfwidth  $\Delta\lambda$  (Videnović et al., 1996), was good enough to obtain local electric field strength with sufficient accuracy. In this paper we extend the possibility of an ad-hoc local electric field and excited hydrogen atoms temperature determination to the usage of third characteristic profile parameter - the intensity ratio between the maximum and the center of the profile,  $I_{max}/I_c$ . This study is confined to the 0 – 20 kV/cm range of electric field strengths and 0.1 – 300 eV interval of excited hydrogen temperatures, which are the conditions that are shown to be typical for the cathode fall region of a glow discharge (see e.g. Videnović et al., 1996; Kuraica et al., 1997).

### 2. THEORY

Following either semiclassical or quantum mechanical theory of the linear Stark effect applied to the hydrogen and hydrogen-like emitter one obtains the known result that each energy level is splitted into equidistant sub-levels (Condon and Shortley, 1977). Therefore, the hydrogen spectral lines consist of numerous components which are polarized either linearly, parallel to the vector of external field  $\vec{F}$  ( $\pi$ -components), or circularly, in the plane perpendicular to  $\vec{F}$  ( $\sigma$ -components). The relative wavelength positions of these components are proportional to the electric field strength and form the characteristic symmetrical Stark patterns (Condon and Shortley, 1977). The fine structure splitting introduces the asymmetry in the Stark patterns of hydrogen lines. To calculate the eigenvectors and eigenvalues of the hydrogen atom, the perturbed Hamiltonian

$$H_n = H_{0n} + q\vec{F} \cdot \vec{R}_n \quad (1)$$

is used, where  $\vec{F} = (0, 0, F)$  is the perturbing electric field pointing along  $OZ$  axis,  $H_{0n}$  is the Hamiltonian of the hydrogen atom including the fine structure and  $\vec{R}_n$  is the projection of the distance operator  $\vec{R}$  on the space of the states with main quantum number  $n$  (no-quenching approximation). The eigenstates of the whole Hamiltonian  $H_n$  are defined by the relationship

$$H_n |v\rangle = E_v |v\rangle. \quad (2)$$

To carry out the numerical calculation, the basis of  $H_{0n}$  eigenstates  $|n, j, l, m_j\rangle$  is used. The evolution operator of the group of states with main quantum number  $n$  for a given configuration of the static electric field  $\vec{F}$  is given by:

$$U_n(t) = \exp\left(-\frac{i}{\hbar} H_n t\right) = \sum_{\nu} P_{\nu} \exp\left(-\frac{i}{\hbar} E_{\nu} t\right). \quad (3)$$

The intensities of the components of the transition  $n \rightarrow n'$  are given by the relationship (Gigosos and González, 1998)

$$I_{nn'} = 2 \operatorname{Re} \operatorname{tr}(\vec{d}_{n'n} \cdot U_n^{\dagger} \vec{d}_{nn'} U_n), \quad (4)$$

where  $\vec{d}_{nn'}$  is the  $nn'$  box of the atom dipole momentum operator that connects the group of states  $n$  with the  $n'$  one. Using (5), one obtains

$$I_{nn'} = 2 \operatorname{Re} \sum_{\nu} \sum_{\nu'} \exp\left[-\frac{i}{\hbar}(E_{\nu} - E_{\nu'})t\right] \operatorname{tr}(\vec{d}_{n'n} \cdot P_{\nu} \vec{d}_{nn'} P_{\nu'}), \quad (5)$$

where  $P_{\nu}$  is the projector on the subspace of states with eigenvalue  $E_{\nu}$ . By using the  $Z$  component of the vector  $\vec{d}_{nn'}$ , the intensities of  $\pi$  transitions are obtained, while  $X$  and  $Y$  components yield the intensities of  $\sigma$  transitions. In the numerical modeling of Stark profiles of hydrogen lines we assumed that plasma broadening effects in the cathode fall region of a glow discharge may be neglected and to the each Stark component we have assigned a Gauss function only which takes into account the instrumental and Doppler broadening (see Videnović et al., 1996). The overall  $\pi$  or  $\sigma$  polarized profile are calculated as the superposition of all components polarized in the appropriate manner.

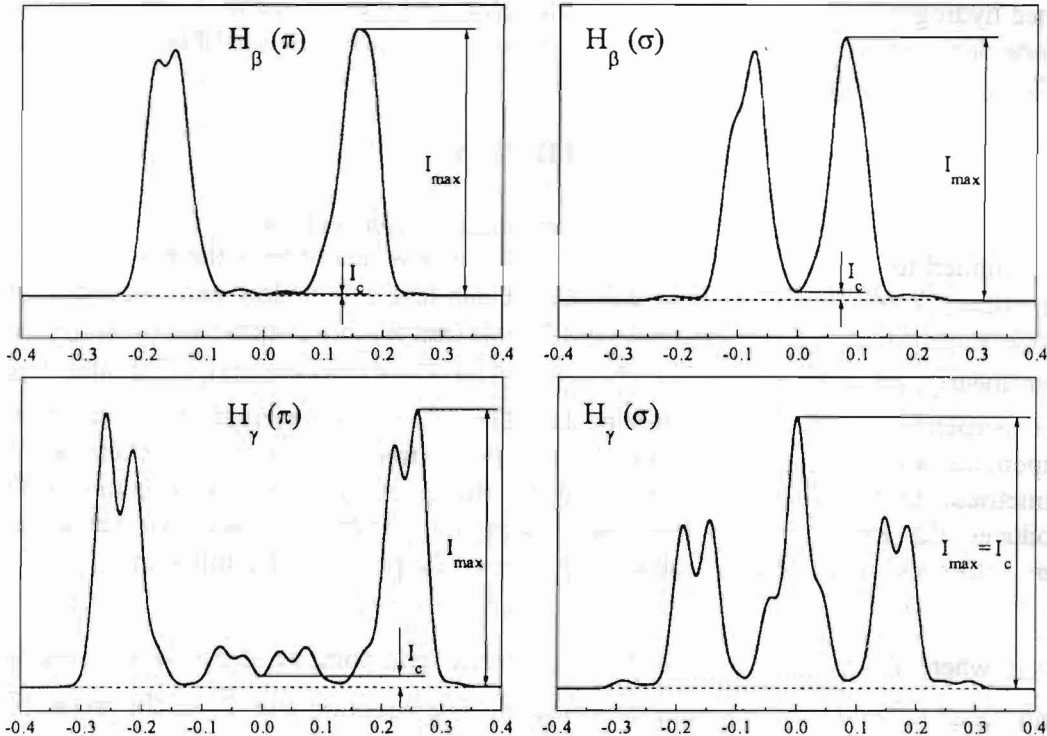


Fig. 1. Polarized Stark  $H_{\beta}$  and  $H_{\gamma}$  profiles, calculated upon the fine structure manifold, for the electric field strength of 12 kV/cm and the excited H-atoms temperature of 1 eV. On the abscissa axes relative wavelengths related to the unperturbed line (in nm) are given. Characteristic maximum  $I_{\max}$  and central  $I_c$  intensities are defined also.

### 3. RESULTS AND DISCUSSION

The examples of the polarized profiles of the hydrogen Balmer  $H_\beta$  and  $H_\gamma$  lines, calculated following the fine structure manifold, for the electric field strength of 12 kV/cm and the excited hydrogen atoms temperature of 1 eV, are shown in Fig. 1. These conditions are selected as being of order typical for the cathode fall region of a glow discharge, see Videnović et al., 1996. For all profiles the intensities  $I_{\max}$  and  $I_c$  are defined. Obviously, for the  $H_\gamma(\sigma)$  profile the ratio between these intensities is not applicable.

The temperature dependencies of the  $I_{\max}/I_c$  ratio for the  $H_\beta(\pi)$ ,  $H_\beta(\sigma)$  and  $H_\gamma(\pi)$  polarized profiles, at several electric field strengths in the range 0 – 20 kV/cm are shown in Figures 2, 3 and 4, respectively. For all profiles, with the temperature increase, the Stark splitting is overlapped with the Doppler broadening and profiles degenerate into single Gaussian, leading to the intensity ratio  $I_{\max}/I_c$  value of 1.

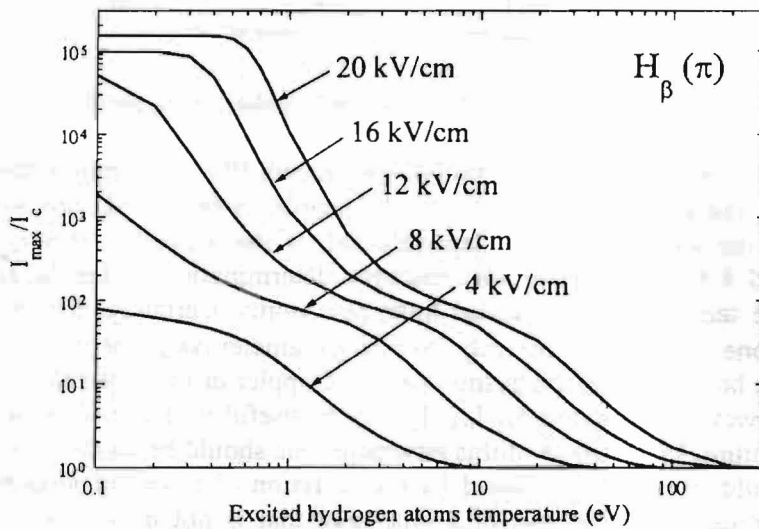


Fig. 2. Temperature dependencies of the intensity ratio  $I_{\max}/I_c$  of the  $\pi$ -polarized  $H_\beta$  profile for several electric field strengths.

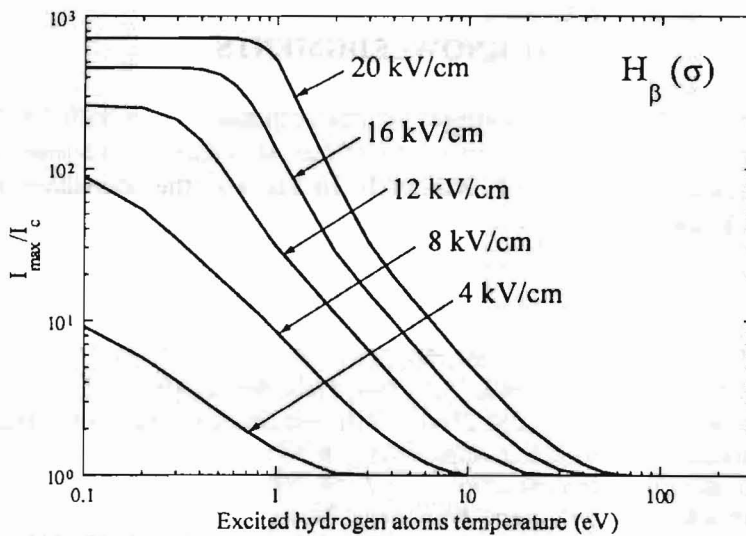


Fig. 3. Same as in Fig. 2, but for the  $\sigma$ -polarized  $H_\beta$  profile

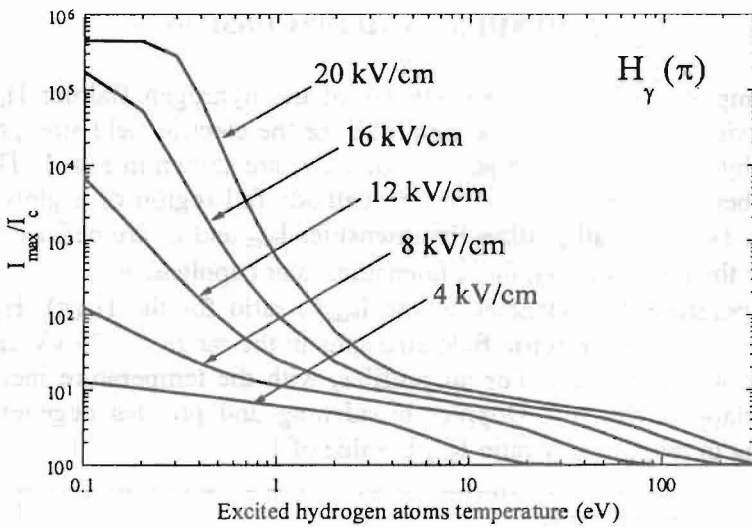


Fig. 4. Same as in Fig. 2, but for the  $\pi$ -polarized  $H_\gamma$  profile.

In conjunction with other line profile parameters (the wavelength distance between maximums  $\Delta\lambda_{pp}$ , see Kuraica et al., 1997, or the profile halfwidth  $\Delta\lambda$ , see Videnović et al., 1996) one may use the temperature dependencies of the intensity ratio  $I_{max}/I_c$  shown in figures 2, 3, and 4 for the ad-hoc spectroscopic determination of the local electric field intensity and the temperature of excited hydrogen atoms. Certainly, for the electric field determination, one should mostly rely to the parameter  $\Delta\lambda_{pp}$ , because it is the least dependent on the broadening mechanisms, such as Doppler or instrumental ones. For known electric field, however, the parameter  $I_{max}/I_c$  may be useful in determination of the excited H-atoms temperature. In the usage of this parameter one should be cautious, bearing in mind that its value could be strongly affected by the emission of so-called *field-free* lines – the lines that are emitted from the part of a discharge that is not under the influence of the external electric field (see e.g. Videnović et al., 1996; Kuraica and Konjević, 1997; Kuraica et al., 1997). These lines are detected at the central wavelength position, making the parameter  $I_{max}/I_c$  inapplicable.

#### ACKNOWLEDGMENTS

The author gratefully acknowledges private communication with Dr. Marco Antonio Gigosos who kindly provided the calculations of fine structure components and intensities. The valuable discussions with Prof. Nikola Konjević and the assistance of Mirjana M. Platiša are also acknowledged.

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