

# A Study of Supersonic Turbulence in Stagnating Plasma

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A. Fruchtman<sup>3</sup>

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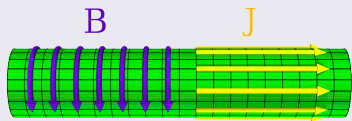
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11th Serbian Conference on Spectral Line Shapes in Astrophysics  
Šabac, Serbia  
August 21–25, 2017

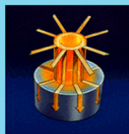
# How Z-pinch works



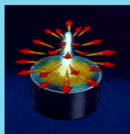
The  $\vec{J} \times \vec{B}$  Lorentz force makes the plasma implode.



**Initiation**



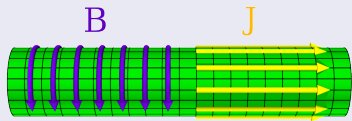
**Implosion**



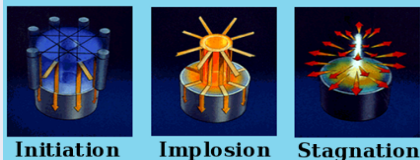
**Stagnation**

Most of the X-ray emission takes place at the stagnation phase.

# How Z-pinch works

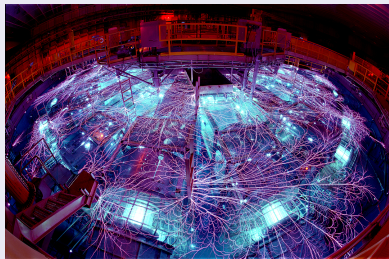


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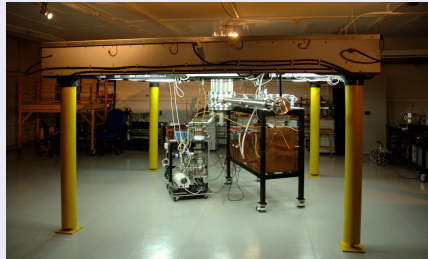


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“Z machine” (Sandia Labs, US)



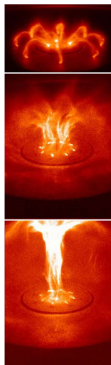
Z-pinch in Weizmann Inst. (Israel)



## Focus: Plasma Jets on Earth

August 2, 2005 • Phys. Rev. Focus 16, 4

A simple arrangement of electric and magnetic fields causes plasma to form shapes reminiscent of the jets generated near supermassive black holes.



Phys. Rev. Lett. 95, 095002 (2005)

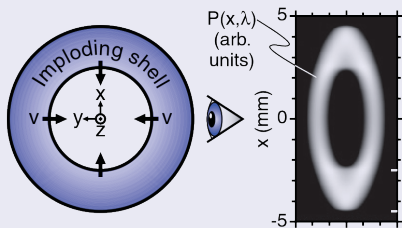
[You et al., 2005]

*“A simple arrangement of electric and magnetic fields causes plasma to form shapes reminiscent of the jets generated near supermassive black holes.”*

Pinching is a naturally occurring phenomenon, but importance of z-pinches as an astrophysical laboratory goes well beyond that.

# Pinches as laboratory astrophysics: im/explosions

## Z-pinch ( $r \sim 1$ cm)



## [Foord et al., 1994]

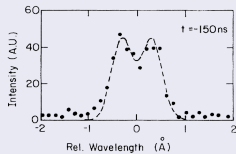
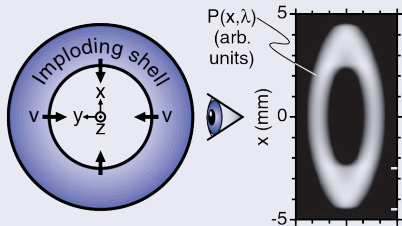


FIG. 2. A typical O III line profile ( $\lambda_0 = 3047.1 \text{ \AA}$  measured radially at  $z = 11$  mm and  $t' = -150$  ns. Redshifted and blueshifted components from each side of the annulus are observed. Doppler shifts correspond to a radial velocity  $\approx 3$  cm/ $\mu$ s.

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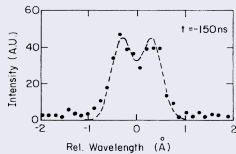
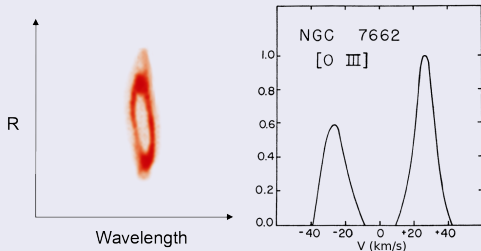


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## NGC 7662 ( $r \sim 1$ ly)

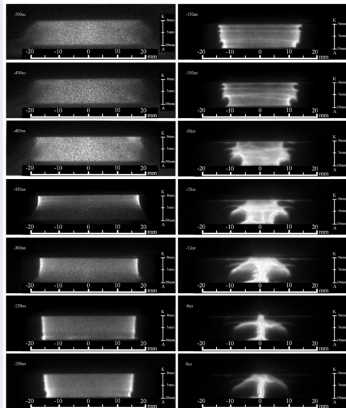


## [Osterbrock et al., 1966]



# Pinches as laboratory astrophysics: instabilities

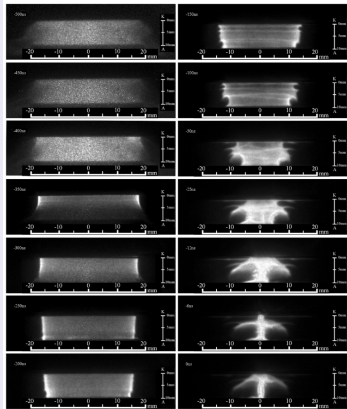
[Osin et al., 2011]



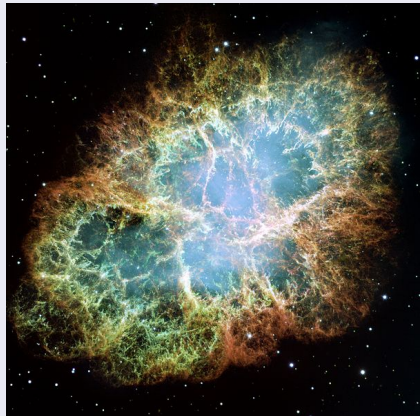
Rayleigh-Taylor instability produces “fingers” and filaments.

# Pinches as laboratory astrophysics: instabilities

[Osin et al., 2011]



NGC 1952 (Crab Nebula)



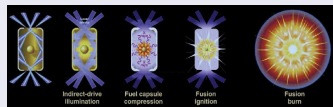
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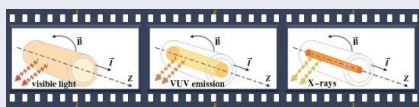
# Energy conversion in imploding plasmas

Imploding plasmas are promising candidates for fusion (NIF, MagLIF) and unique sources of intense x-ray radiation (z-pinches).

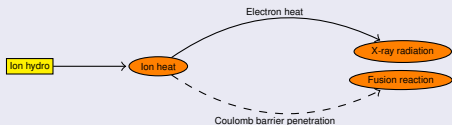
## NIF



## Z-pinch



## Conversion of hydrodynamic ion motion



Thus, one needs to measure ion  $T_i$ . It is also important to know the hydro energy.

## Principal difficulty:

The Doppler broadening (also, neutron spectrum) gives information only on the **total** ion velocity distribution  $\rightarrow T_i^{\text{eff}} \geq T_i$ .

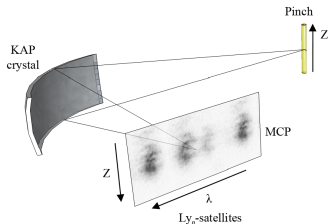
# Importance of distinguishing between $T_i^{\text{eff}}$ and $T_i$

Assuming  $T_i = T_i^{\text{eff}}$  may result in crucially misinterpreted data.

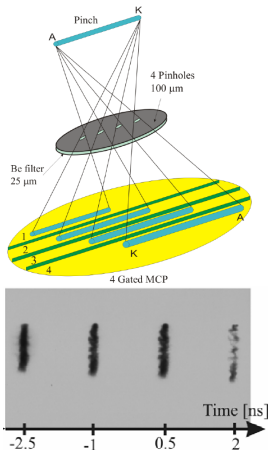
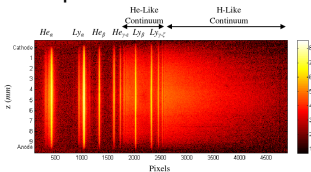
At WIS, we have succeeded developing advanced diagnostics capable of telling  $T_i^{\text{eff}}$  and  $T_i$  apart.

# Diagnostics setup

Three time-resolved data sources: spectra of Ne Ly- $\alpha$  dielectronic satellites, gated x-ray pinhole imaging, and an absolutely calibrated photo-conductive detector (PCD) sensitive to  $\hbar\omega \gtrsim 700$  eV.



In addition, time-integrated wide spectrum is taken.



# Plasma parameters inferred

Assuming the plasma is uniform, we obtain

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$T_i$  is inferred from the data above, plus using either

- Detailed energy-balance analysis [Kroupp et al., 2011, Maron et al., 2013]; or
- Effect of  $\Gamma_{ii}$  on Stark lineshapes of high- $n$  transitions [Alumot et al., 2017] (in preparation).



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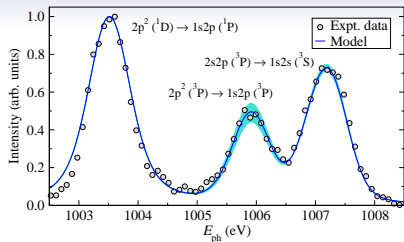
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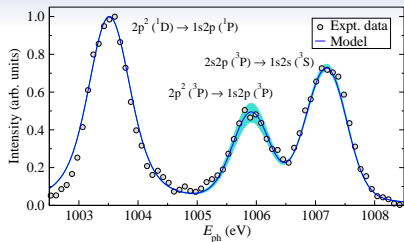
This modeling described all the data very well, within 1 – 2 std. dev.

# Determination of $n_e$ and $T_i^{\text{eff}}$

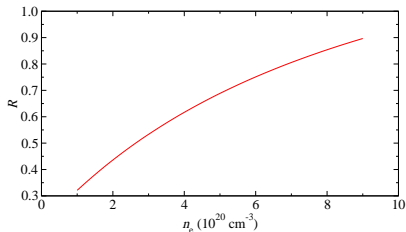


Widths and intensities of the Ne Ly- $\alpha$  satellites allow for determining  $T_i^{\text{eff}}$  and  $n_e$ , respectively. In this example,  $T_i^{\text{eff}} = 1200 \text{ eV}$  and  $n_e = (5 \pm 1) \times 10^{20} \text{ cm}^{-3}$ .

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$I_{2p^2(^3P) \rightarrow 1s2p(^3P)} / I_{2s2p(^3P) \rightarrow 1s2s(^3S)}$  intensity ratio  $R$  is sensitive to  $n_e$  but practically independent of  $T_e$  [Seely, 1979, Kroupp et al., 2007].

# Determination of $T_i$ during z-pinch stagnation

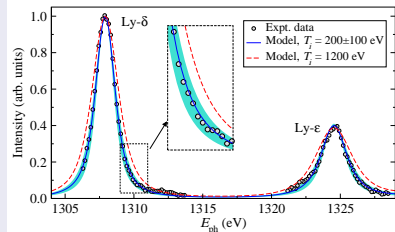
Two methods have been used:

## 1. Detailed energy-balance analysis

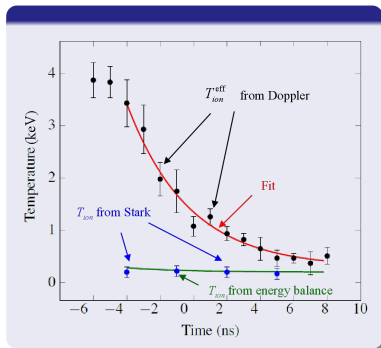
$$\frac{T_i - T_e}{\tau_{ie}} = \frac{dT_i^{\text{eff}}}{dt}$$

[Kroupp et al., 2011]

## 2. Effect of $\Gamma_{ii}$ on Stark lineshapes



[Alumot et al., 2017] (in preparation)



Both methods give the same, consistent results.

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Were supersonic turbulence present, it would imply substantial nonuniformity in quantities such as the density. However, the previous analysis assumed a uniform plasma.

# New analysis

The data need to be re-analyzed assuming a physically sound model of turbulence. [Kroupp et al., 2017]

Instead of  $n_e = n_e^0 = \text{const}$ , there is now a probability distribution function (PDF)  $P(n_e)$  – actually,  $P(t, z; n_e)$ .

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Let us switch to dimensionless quantity

$$\xi \equiv n_e/n_e^0; \int P(\xi) d\xi = 1.$$

The average density is

$$\langle n_e \rangle = n_e^0 \int \xi P(\xi) d\xi.$$

(note that  $\langle n_e \rangle \neq n_e^0$ ).

Assuming the collisional-radiative equilibrium is established much faster than the hydromotion, the intensity of a spectral line (or continuum radiation) is [Stamm et al., 2017]

$$\langle I \rangle = \int \alpha(\vec{r}) d^3 r = \pi r_{\text{pl}}^2 \ell \int \alpha(\xi) P(\xi) d\xi,$$

where  $\alpha \propto \xi^2$  is the local plasma emissivity, and  $r_{\text{pl}}$  and  $\ell$  is the radius and length of the plasma segment, respectively.

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## New analysis (cont.)

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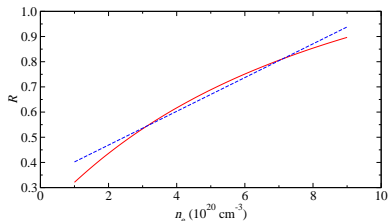
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$$\left(1 - \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}\right) \left(\frac{r_{\text{pl}}^0}{r_{\text{max}}}\right)^2 \leq \int \xi^2 P(\xi) d\xi \leq \left(1 + \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}\right) \left(\frac{r_{\text{pl}}^0}{r_{\text{min}}}\right)^2.$$

## Density determination:



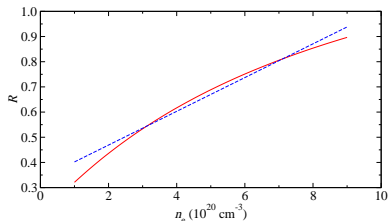
Use linearization

$R \approx R^0 + a_R(n_e/n_e^0 - 1)$ , so

$$\langle R \rangle = R^0 + a_R \frac{\int (\xi - 1) \xi^2 P(\xi) d\xi}{\int \xi^2 P(\xi) d\xi}.$$



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The measured quantity  $R_{\text{expt}}$  is known – **and should remain** – within its error bars, i.e.,  $\langle R \rangle = R_{\text{expt}} = R^0 \pm \delta R$ . Therefore,

$$1 - \frac{\delta R}{a_R} \leq \frac{\int \xi^3 P(\xi) d\xi}{\int \xi^2 P(\xi) d\xi} \leq 1 + \frac{\delta R}{a_R}.$$

To summarize:

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Once  $P(\xi)$  is determined, the model plasma radius is corrected:

$$r_{\text{pl}} = r_{\text{pl}}^0 / \sqrt{\langle \xi^2 \rangle} .$$

## Constraints on $P_V(\xi_V)$ and $\beta$

The last step is to use the volumetric density distribution:

$$\int P_V(\xi_V) d\xi_V = 1, \quad \int \xi_V P_V(\xi_V) d\xi_V = 1.$$

Introduce  $\beta \equiv \xi/\xi_V = \langle n_e \rangle/n_e^0$ ; so  $\langle \xi^k \rangle = \beta^k \langle \xi_V^k \rangle$ .

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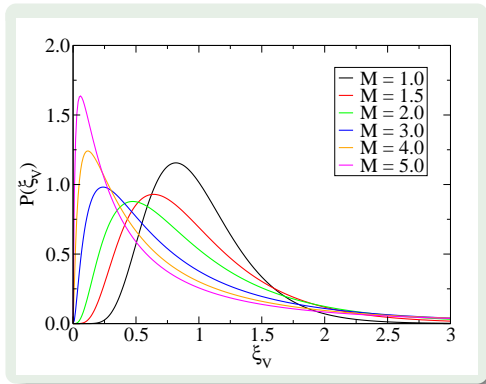
$$\sqrt{\frac{1 - \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{max}}} \leq \beta \leq \sqrt{\frac{1 + \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{min}}}$$

$$\left(1 - \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle} \leq \beta \leq \left(1 + \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle}$$

$$r_{\text{pl}} = r_{\text{pl}}^0 / (\beta \sqrt{\langle \xi_V^2 \rangle})$$

Volumetric PDF of [Hopkins, 2013]:

$$P_V(\xi_V) d\xi_V = \frac{I_1(2\sqrt{\lambda\omega(\xi_V)})}{\exp[\lambda + \omega(\xi_V)]} \sqrt{\frac{\lambda}{\theta^2\omega(\xi_V)}} \frac{d\xi_V}{\xi_V}$$



Here,

$$\lambda \equiv (1 + \theta)^{3/2} \ln(1 + M_c^2) / 2\theta^2$$

$$\omega(\xi_V) \equiv \lambda / (1 + \theta) - \ln(\xi_V) / \theta$$

$$\theta \approx 0.05M_c$$

Compressive Mach number

$$M_c = bM, \quad b \approx 0.4$$

$I_1$  – modified Bessel function  
of the first kind

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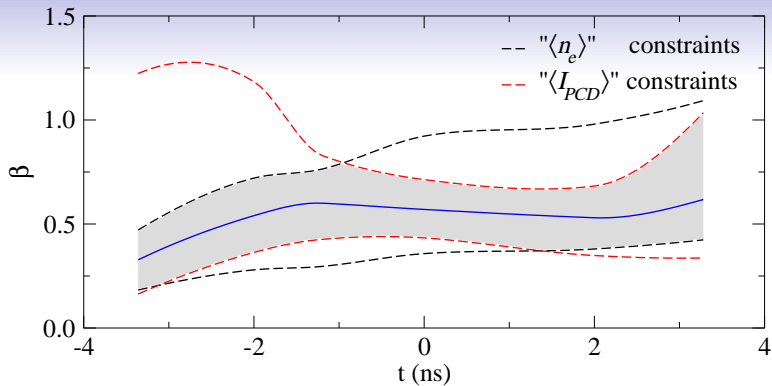
Finally:

$$\sqrt{\frac{1 - \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{max}}} \leq \beta \leq \sqrt{\frac{1 + \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{min}}}$$

$$\left(1 - \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle} \leq \beta \leq \left(1 + \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle}$$

$$r_{\text{pl}} = r_{\text{pl}}^0 / (\beta \sqrt{\langle \xi_V^2 \rangle})$$

# Results: model mean density and plasma radius

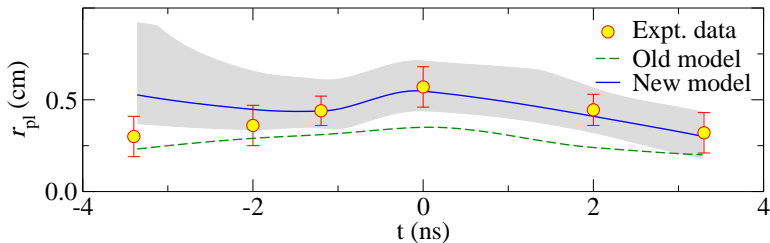


The mean plasma density is inferred to be lower by a factor  $\sim 2$ .

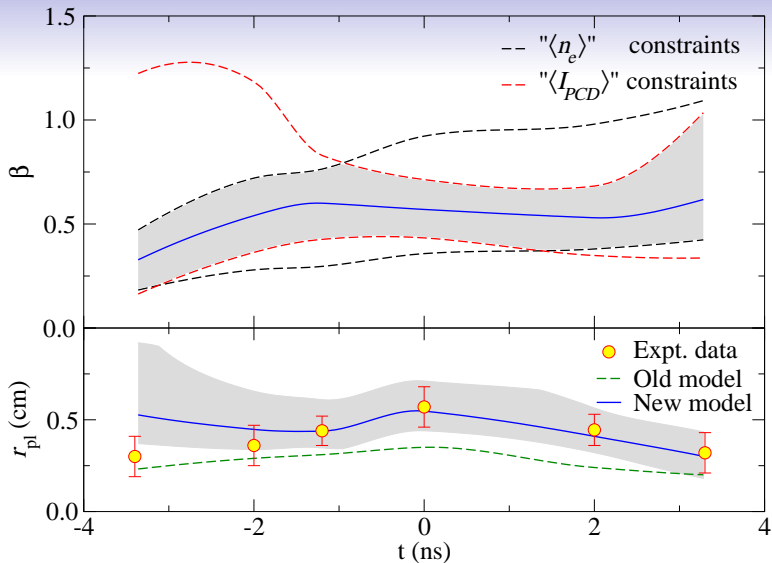


# Results: model mean density and plasma radius

The corrected plasma model radius fits the data better.



# Results: model mean density and plasma radius



The other plasma parameters ( $T_e$ ,  $T_i$ , and  $T_i^{\text{eff}}$ ) remain unaffected.

## Results in a wider scientific context

In addition to better understanding of z-pinch stagnation plasmas, a crucial question arises:

Is the supersonic turbulent hydromotion generated and carried along during the implosion phase?

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Is the supersonic turbulent hydromotion generated and carried along during the implosion phase?

If yes (and we have preliminary results confirming it), then z-pinchs represent a test bed for:

- a recently proposed novel fast ignition scheme [Davidovits and Fisch, 2016] for inertial confinement;
- astrophysical phenomena, such as molecular cloud dynamics, star formation efficiency, the core mass/stellar initial mass function, and more.

# Conclusions

- Inferred  $T_i \ll T_i^{\text{eff}}$  at z-pinch stagnation, with  $\text{Re} \sim 10^5$  and observing no large-scale plasma non-uniformities, strongly hints at turbulence; supersonic one ( $M \sim 1 - 2$ ).

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- This turbulent-plasma model is not only consistent with the observations, it improves the agreement with them ( $r_{\text{pl}}$ ).
- Beyond aiding our understanding of z-pinches, we hope this study can provide fertile ground for dealing with related problems of astrophysical interest.

Thank you!

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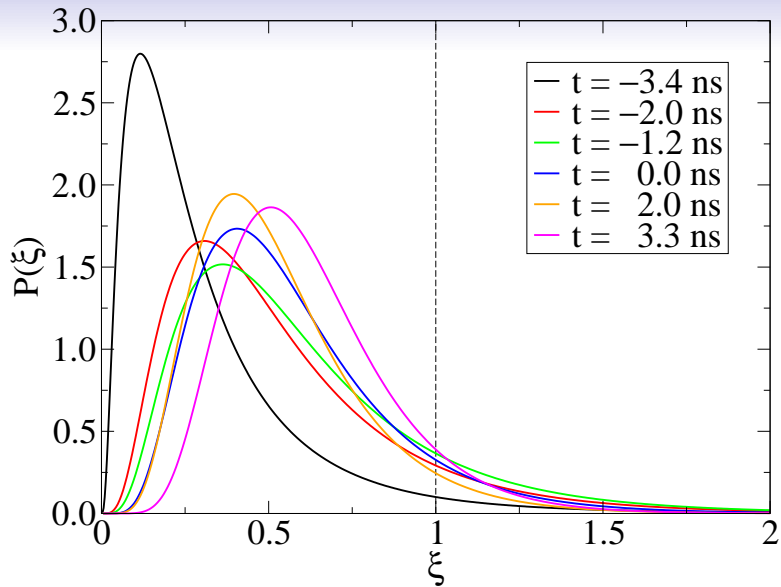


# Data summary

The experimental data [Kroupp et al., 2011] relevant for the analysis presented; the plasma parameters assumed for  $(r_{\text{pl}}^0, n_e^0, T_e)$  and inferred from  $(T_i, M, \text{Re})$  the *uniform*-plasma modeling; the calculated isothermal turbulence parameters, volumetric density factor  $\beta$  and respectively corrected plasma electron density and radius. Units are as follows: all radii are in mm, all temperatures are in eV, and densities are in  $10^{20} \text{ cm}^{-3}$ .

t (ns)	Experimental data					Uniform plasma						Isothermal turbulence						
	$\delta R$	$I_{\text{PCD}}$ (GW)	$r_{\text{min}}$	$r_{\text{max}}$	$T_i^{\text{eff}}$	$r_{\text{pl}}^0$	$n_e^0$	$T_e$	$T_i$	$M$	$\text{Re}$	$\theta$	$\sigma_{s,V}^2$	$\langle \xi_V^2 \rangle$	$\langle \xi_V^3 \rangle$	$\beta$	$n_e^{\text{turb}}$	$r_{\text{pl}}^{\text{turb}}$
-3.4	0.15	$0.35 \pm 0.3$	0.19	0.41	3000	0.23	6.0	120	250	2.4	$8.1 \times 10^4$	0.048	0.70	1.84	5.77	0.32	1.9	0.53
-2.0	0.15	$2.0 \pm 1.0$	0.25	0.47	2100	0.29	6.0	175	230	1.7	$6.9 \times 10^4$	0.034	0.40	1.44	2.86	0.54	3.2	0.45
-1.2	0.15	$3.8 \pm 1.1$	0.36	0.52	1800	0.31	6.0	190	210	1.6	$7.7 \times 10^4$	0.032	0.36	1.39	2.60	0.60	3.6	0.44
0.0	0.15	$6.5 \pm 0.7$	0.46	0.68	1300	0.35	6.0	185	200	1.3	$8.9 \times 10^4$	0.026	0.25	1.26	1.96	0.57	3.4	0.55
2.0	0.15	$3.6 \pm 1.0$	0.36	0.53	900	0.24	6.0	155	180	1.2	$7.4 \times 10^4$	0.024	0.21	1.22	1.80	0.53	3.2	0.41
3.3	0.15	$2.3 \pm 0.9$	0.21	0.43	720	0.20	6.0	140	180	1.0	$5.1 \times 10^4$	0.020	0.15	1.16	1.53	0.62	3.7	0.30

# Results: turbulent density PDF's





# Is turbulence in this stagnating plasma isothermal?

Compare  $v_{\text{flow}}$  to a thermal conduction velocity (following [Zeldovich and Raizer, 1967]),

$$v_{\text{cond}} = \frac{L_h}{\tau_{\text{cond}}} \approx 4 \times 10^{21} \frac{\zeta(\langle Z_i \rangle)}{(\langle Z_i \rangle + 1) \lambda_{ei} n_e L_h} T^{5/2},$$

where  $L_h$  is a length scale ( $L_h \sim r_{\text{pl}}$ ),  $\lambda_{ei}$  is the Coulomb logarithm, and  $\zeta(8.5) \approx 2.7$ ;  $T$  is in units of eV,  $n_e$  in  $\text{cm}^{-3}$ , and  $L_h$  in cm.

When  $v_{\text{cond}}/v_{\text{flow}} \gg 1$ , isothermality is expected.

$v_{\text{cond}}/v_{\text{flow}} \sim 2$  for  $L_h \sim r_{\text{pl}}$  at  $t = -3.4$  ns, and  $\sim 6$  for later times.

An accurate determination of the degree of isothermality would require detailed simulations. The inferred  $T_e$  will also need to be reconsidered, since the emissivity depends on  $T_e$  strongly.

Interestingly, in Sandia Z experiments, where  $T_e$  and  $\langle Z_i \rangle$  reach higher values,  $v_{\text{cond}}/v_{\text{flow}} \gg 1$ , thus the assumption of turbulence isothermality should be fully justified.