Line Shapes in a Plasma affected by Nonlinear Wave Collapse

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Outline

1. Introduction
2. Strong Langmuir turbulence and wave packet collapse
3. Line shape model
4. Results
5. Conclusion

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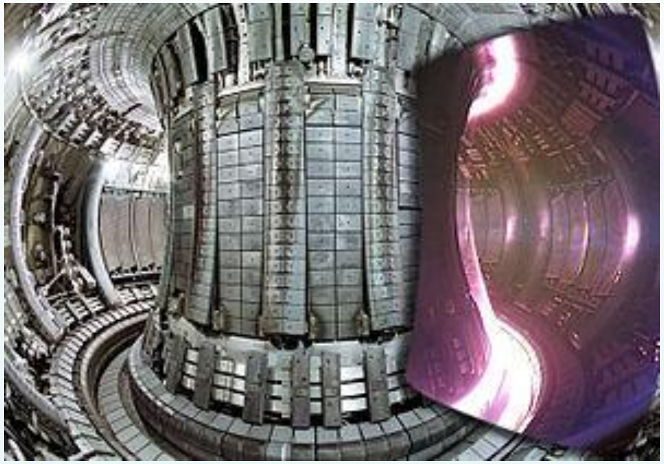
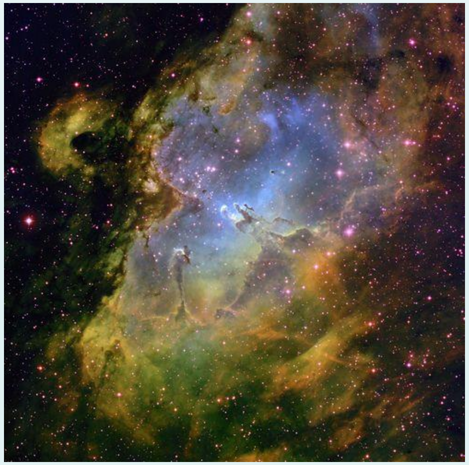
Modeling of plasma radiative properties

-Radiative properties for plasma codes:
Fast and accurate (edge codes, astrophysics)
-Radiative transfer, plasma diagnostic

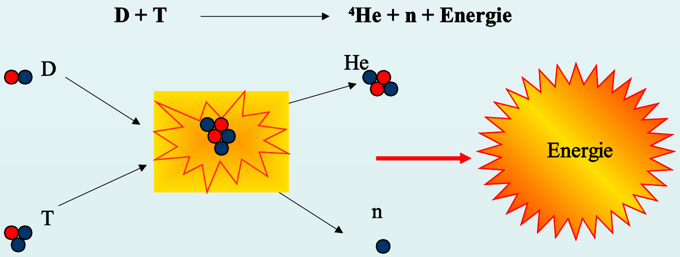
- Stark broadening
- Effect of turbulent plasmas



tokamak JET

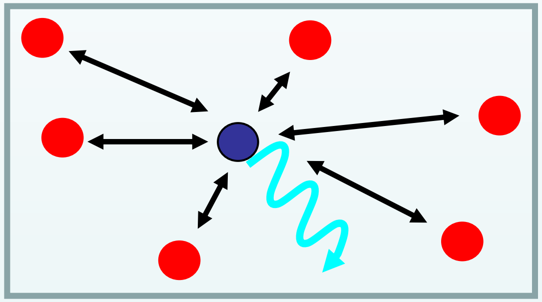


Astrophysics



Fusion

Modeling of plasma radiative properties



Particles

Kinetic theory

Ab initio simulations

Simulation of a large number of particles, coupled to a numerical integration of the Schrödinger equation

Stochastic process

Used for line shapes for conditions that do not allow the use of impact or static approximations

-Equilibrium plasma (Stark broadening)

-Turbulent plasmas

Plasma turbulence

Many different phenomena

Interstellar turbulence : gas velocities are not of pure thermal origin

Fluid turbulence, complex formalism beyond MHD

Laboratory, fusion plasmas : Tokamak strongly affected by turbulent transport. Modeling uses fluid and kinetic theory

This work is restricted to unmagnetized plasmas, and to the possible effects of collectives plasma waves on a line shape

We examine the conditions of **strong Langmuir turbulence** which appear if the plasma is coupled to an energy source (beam of particles)

e.g. Broad hydrogen lines in a tokamak during a disruption

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Strong Langmuir turbulence

Three linear waves in a plasma:

1-Electronic wave at the plasma frequency $\omega_p = \sqrt{\frac{N_e e^2}{m\varepsilon_0}}$

2-Ion acoustic wave involve density fluctuations, they have a constant velocity c_S (plasma sound speed)

3-Electromagnetic waves

The amplitude of waves grows in presence of a beam of particles

Nonlinear coupling of waves 1-2-3 is described by the Zhakarov equations or by numerical simulations

The physical properties of the plasma are changed :

We enter in the **strong Langmuir turbulence** regime

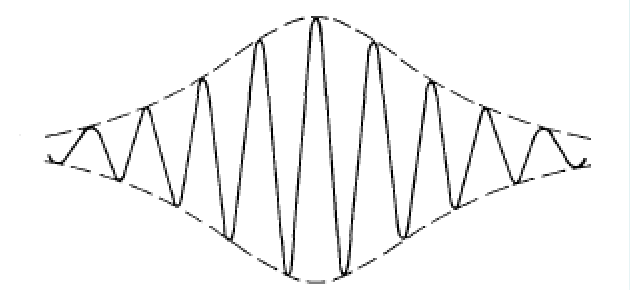
The birth of wave packets

-Density fluctuations create low density regions
-The plasma index of refraction *n* increases in low density regions

e.g. for electromagnetic waves $n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$

-Wave packets localize and grow in such low density regions with a high refractive index

-Modeling: Zhakarov equations reduce to the nonlinear Schrödinger equation in the adiabatic approximation
One dimensional solution : **stable soliton**



$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} + c_S^{-2}|E|^2 E = 0$$

Wave packet collapse

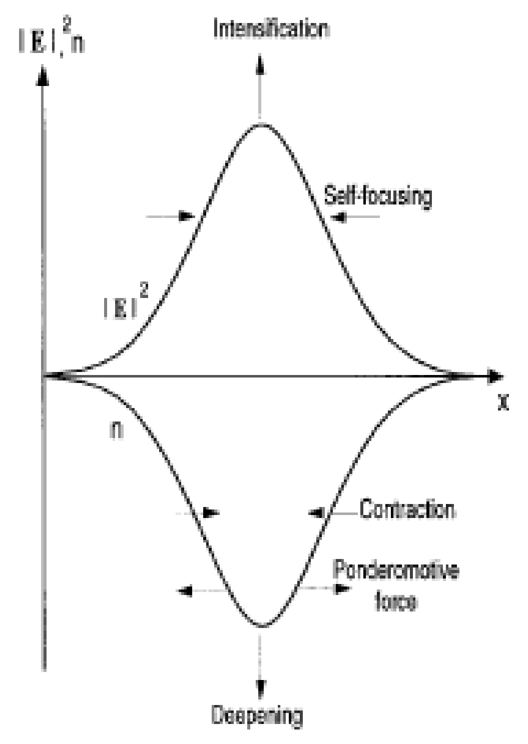
In 2 or 3 D simulations, the wave packet is **unstable**

The nonlinear ponderomotive force acts in a direction opposed to the gradient of the field

$$\overrightarrow{F_{NL}} = -\frac{\omega_p^2}{\omega^2}\vec{\nabla}\frac{\langle \varepsilon_0 E^2 \rangle}{2}$$

The wave packet moves a part of the plasma out of the region of maximum field. In that region, one observes a density depression, and a further increase of the index of refraction

Nonlinear dynamics : the electric field grows to values 100 to 1000 Holtsmark field $E_0=e/r_0^2$



Wave collapse

Wave packet cycle, spatial structure

2 and 3 D simulations reveal

-The existence of a wave packet cycle:

wave packets form, collapse, dissipate, then reform

For plasmas with $T \approx 1$ eV, $10^{11} < N_e < 10^{13}$ cm^{-3},

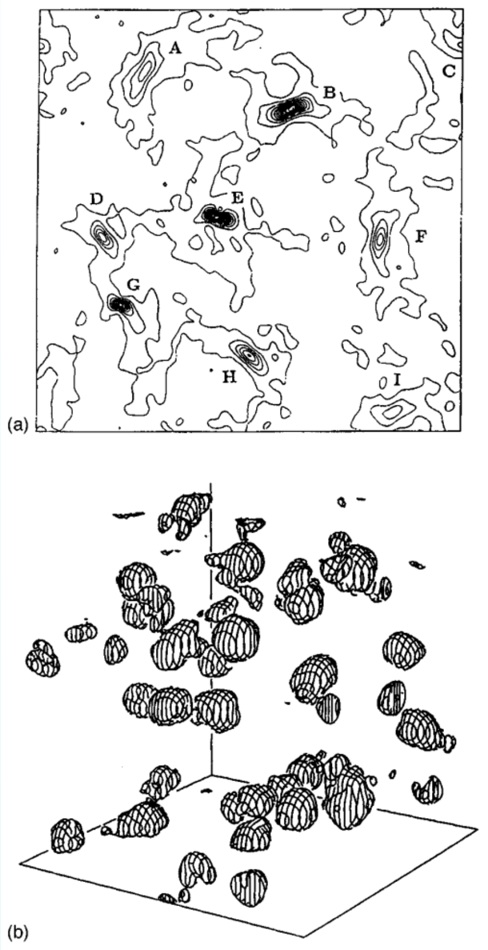
the time for a cycle in the range 30-70 ω_p^{-1}

-The spatial structure of localized wave packets:

Dense packing, mean interpacket separation about 2 to 3 times of the packet length scale

Many wave packets on a line of sight

2 D



3 D Contours of high wave energy

Strong Langmuir turbulence: when and where?

-Ratio *W* of the wave energy density to the plasma energy density

$$W = \frac{\varepsilon_0 \left| \vec{E} \right|^2}{4 N_e k_B T_e}$$

There is a threshold in *W* depending on the plasma conditions

-Wave collapse/strong Langmuir turbulence are thought to exist in a huge range of conditions:

over 23 orders of magnitude of N_e, 4 orders in T, 15 orders in E !

Planetary foreshocks, Auroral regions, ionosphere,

electron beams, laser plasma, fusion plasmas

Many experimental signatures, what about spectral line shapes?

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Line shape with Langmuir turbulence

Studies started in the sixties, Baranger & Mozer, Phys. Rev. **123**, 25 (1961), predicts the observation of satellites at ω_p

Seventies: Oks & Sholin, Bakshi & Kalman

Satellites and some extra broadening effect, confirmed since by a few additional works

Effect of strong Langmuir turbulence on line shapes?

Models: sum of plane waves oscillating at a frequency near ω_p, use of a stochastic process

Line shape with strong Langmuir turbulence

Model for the electric field felt by an atom near to a wave packet

$$\vec{E}(t) = \left\{ \begin{array}{l} \vec{E}_1 \cos(\omega_p t + \varphi_1) S_1(t), \quad 0 \le t \le t_1 \\ \vec{E}_2 \cos(\omega_p t + \varphi_2) S_2(t), \quad t_1 \le t \le t_2 \\ \vdots \\ \vec{E}_n \cos(\omega_p t + \varphi_n) S_n(t), \quad t_{n-1} \le t \le t_n \\ \vdots \end{array} \right\}$$

Renewal stochastic process, with for each jump a new electric field direction and phase.

We can choose:

-the envelope functions $S_j(t)$ and their shape in $\tau_i = t_i - t_{i-1}$

-the probability density function (PDF) for the modulus E

-the waiting time distribution (WTD) between two sucessive jumps

Renewal process for the electric field

-A log-normal PDF for the electric field (Sattin *et al*. 2004)

$$P(E) = \frac{1}{E\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(E))^2}{2\sigma^2}\right)$$

-An exponential WTD for the jumping times $w(t)=\nu \exp(-\nu t)$

The jumping frequency ν is taken equal to the inverse of the average duration of a wave packet cycle (Robinson 1997)

-A Lorentzian envelope functions $S_j(t)$ with a time width ΔT_L having a constant ratio with the step duration τ

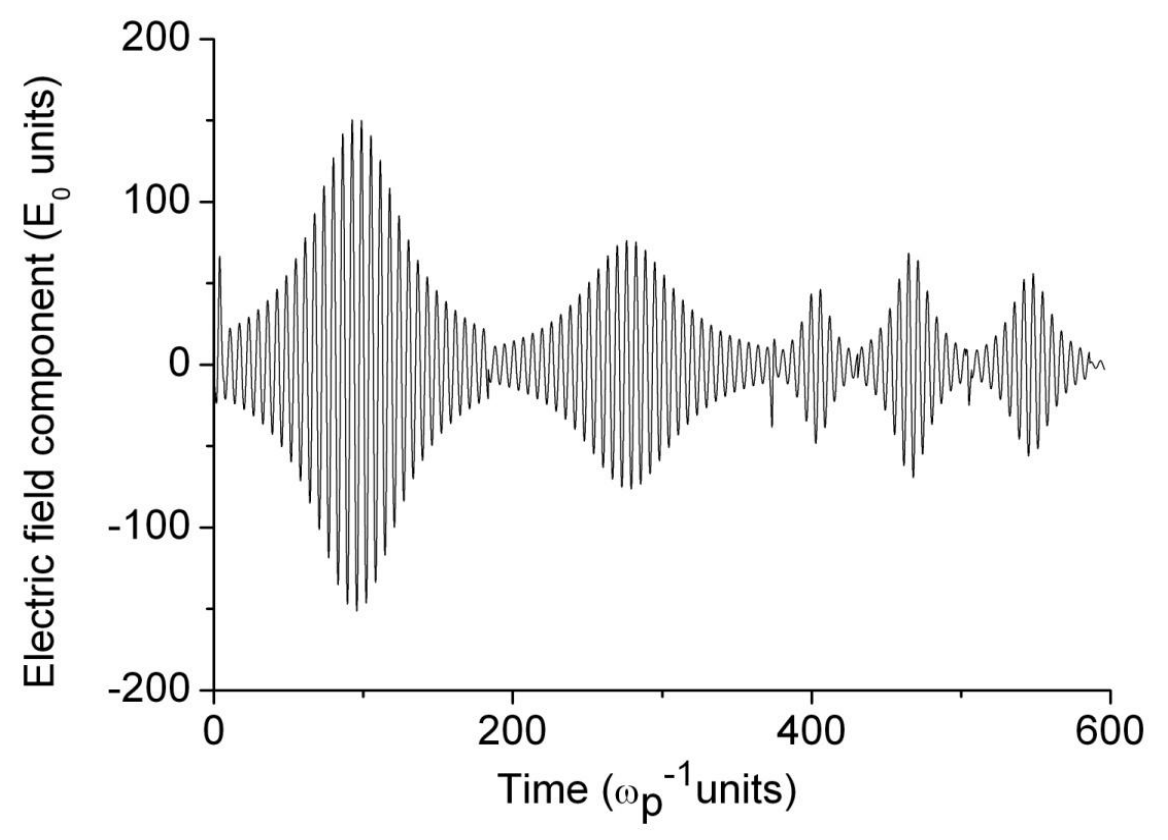
-We use a **simulation** of the stochastic renewal process

Electric field history

Average peak field 100 E_0, jumping frequency $\nu=\omega_p/50$

plasma T= 10^4 K, $N_e=10^{13}$ cm^{-3}

Lorentzian envelope functions $S_j(t)$ with a time width ΔT_L 40% of τ



Calculation of the dipole autocorrelation function

$$C(t) = \mathrm{Tr}\left\langle \rho\, \vec{D}(0)\, U^{+}(t)\, \vec{D}(0)\, U(t) \right\rangle_{av}$$

D is the emitter dipole, U(t) the evolution operator is obtained by solving the Schrödinger equation

$$i\hbar \frac{dU}{dt}(t) = \left(H_0 + V(t)\right) U(t)$$

This is done numerically for a history of $\vec{E}(t), \quad V(t) = -\vec{D}.\vec{E}(t)$
The dipole autocorrelation function (DAF) is obtained after an average over all configurations of the turbulent Langmuir field
In the following we average over 10^4 field histories

Stark broadening: the line shape

Fourier transform of the dipole autocorrelation function (DAF)

$$L(\omega) = \frac{1}{\pi} \operatorname{Re} \int_0^\infty C(t) e^{i\omega t} dt$$

Different calculations are possible

-The single effect of strong Langmuir turbulence (pure Langmuir)

-The effect of equilibrium Stark broadening (pure Stark)

-The result of a convolution of the two previous (full profile)

Calculations for the hydrogen Ly$_\alpha$ line

Ly$_\alpha$ line without fine structure is rapidly calculated with computer simulations

Not well suited for diagnostic, but line shapes calculated in the center of mass for radiative transfer studies in fusion (Rosato et al. 2010)

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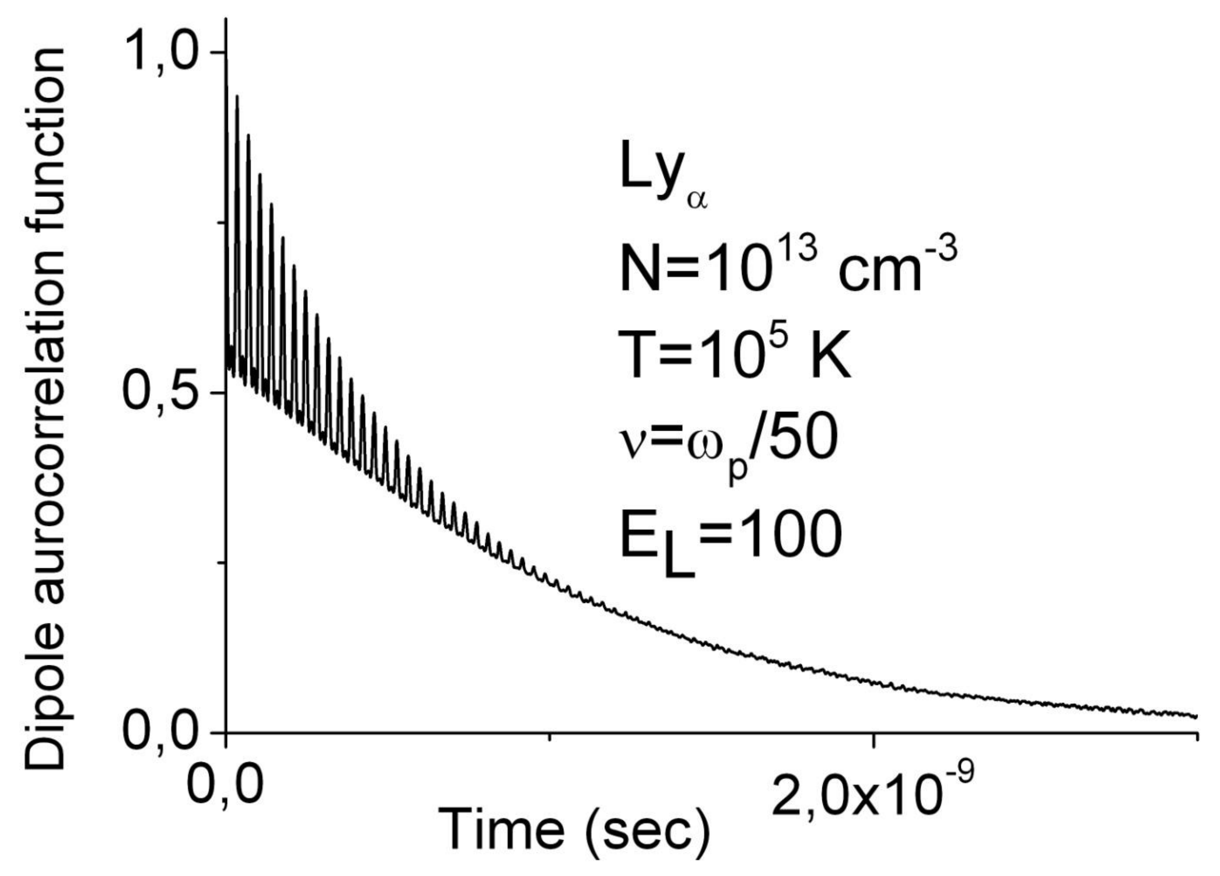
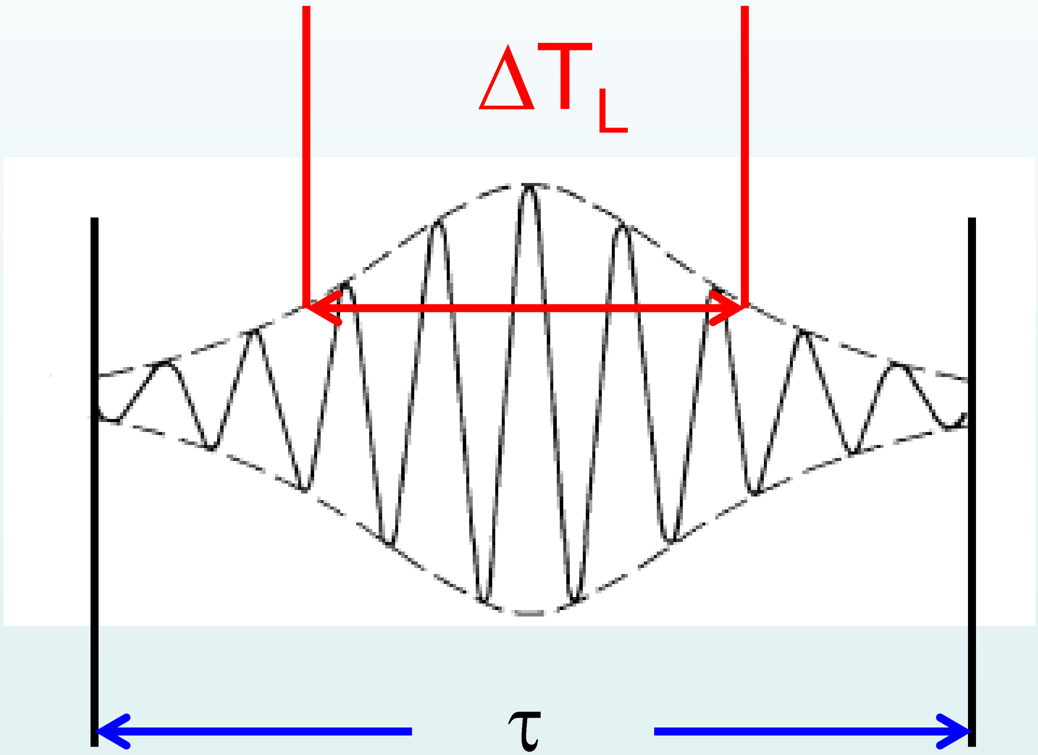
Ly_α dipole autocorrelation function



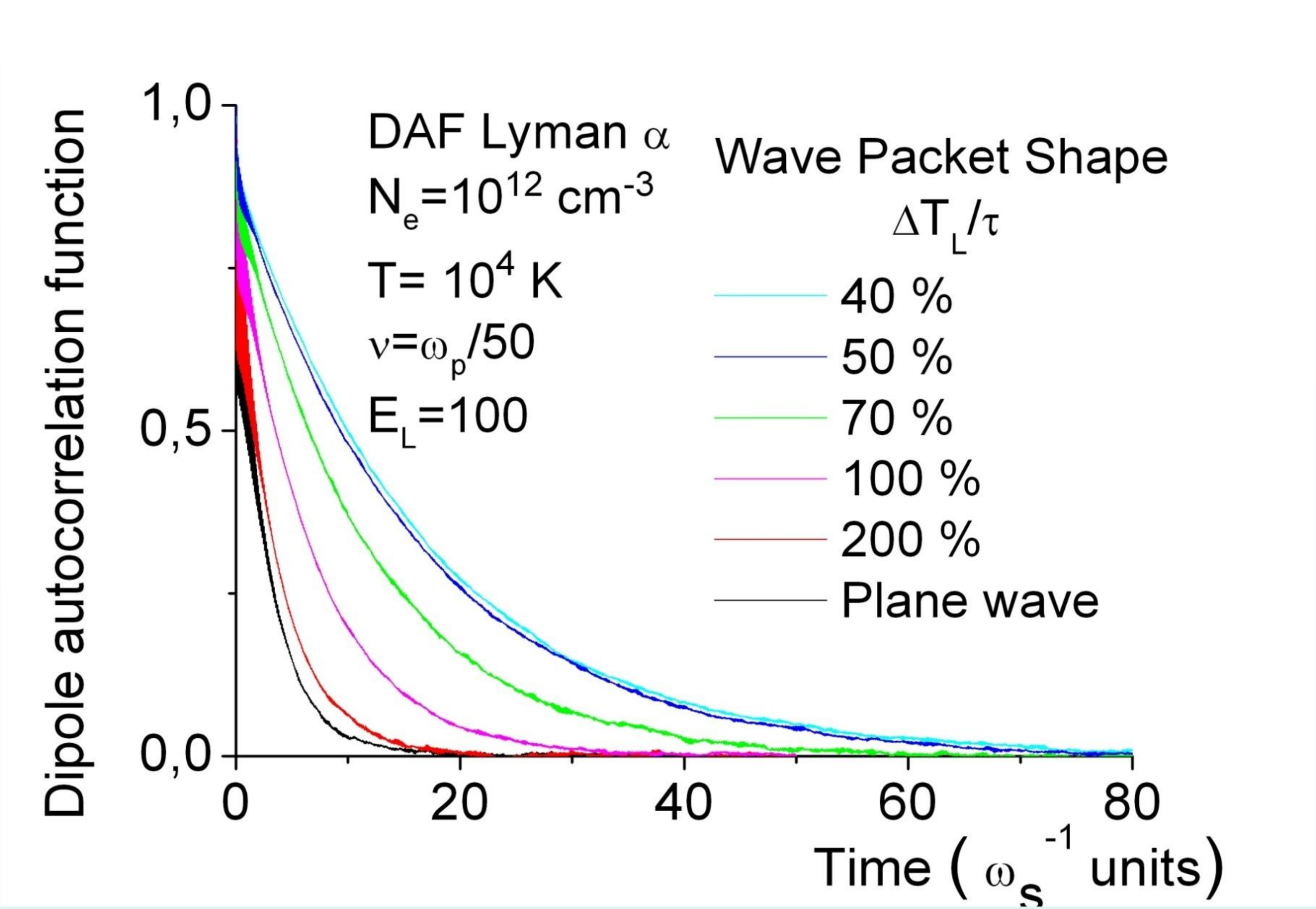
Fast oscillation at ω_p

Ly_{α} DAF, effect of the envelope shape



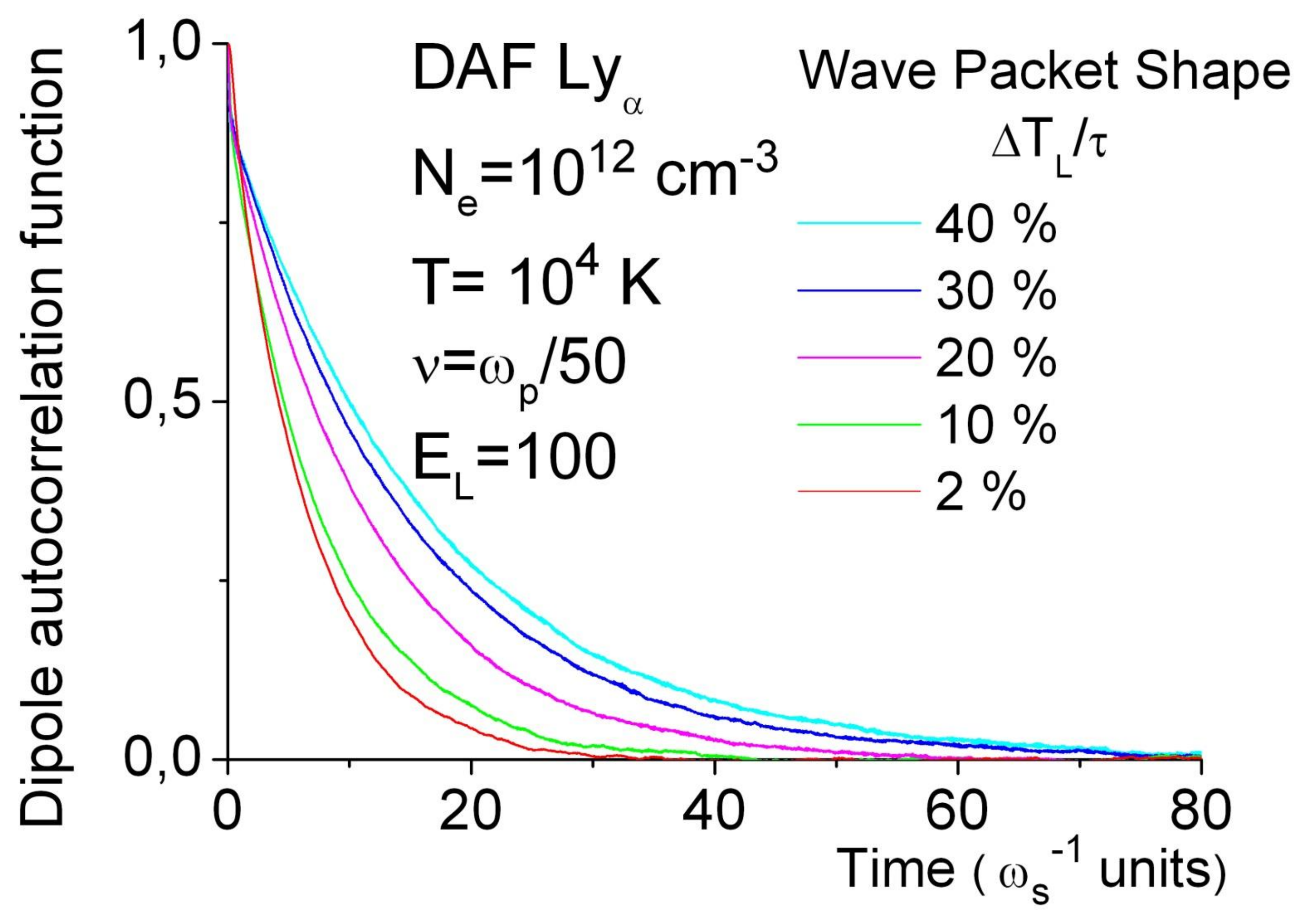
Ratio of the Lorentzian half width ΔT_L to the duration τ of a step (here about 50 %)

Ly_α DAF, effect of the envelope shape



For weakly peaked shapes, decay decreases with decreasing $\Delta T_L/\tau$

Ly_α DAF, effect of the envelope shape

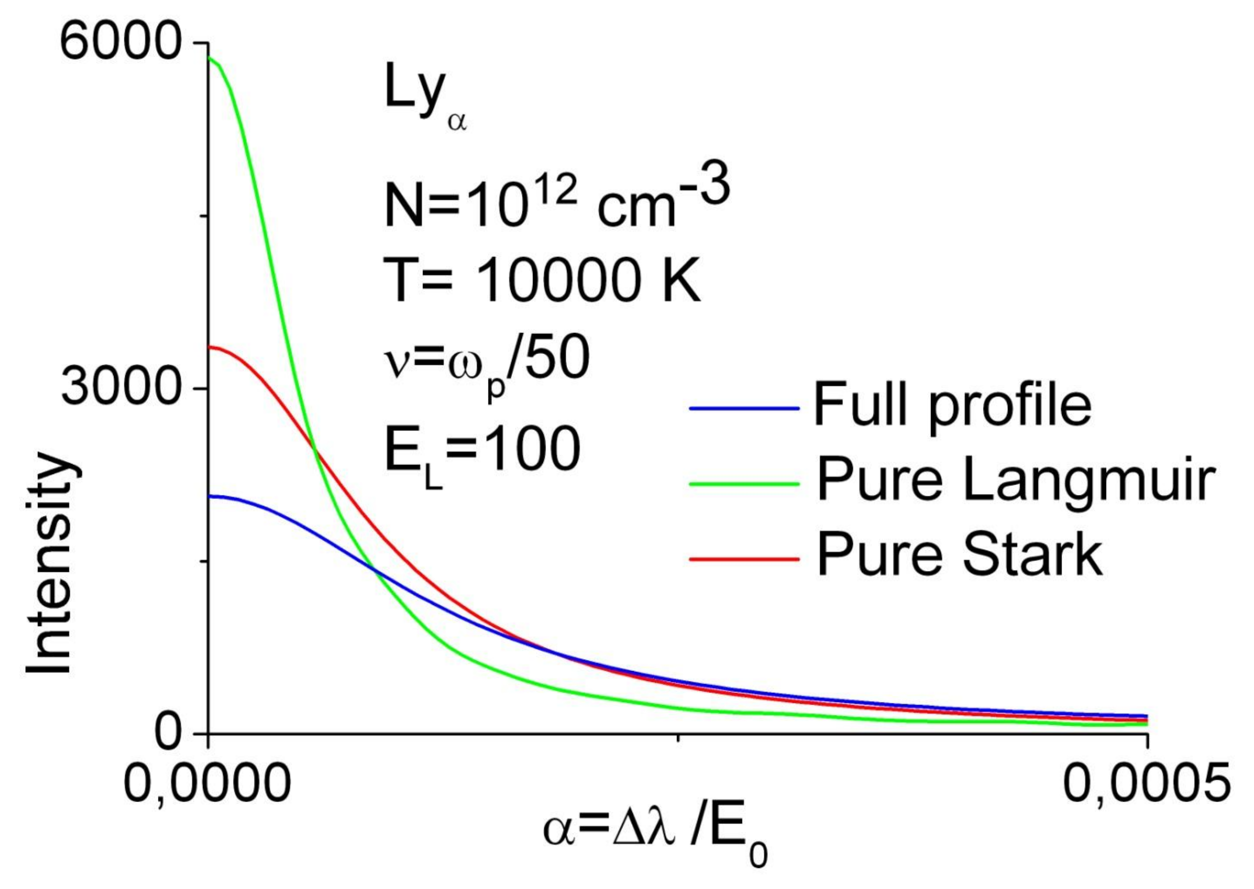


For well peaked shapes, decay increases with decreasing $\Delta T_L/\tau$

In the following we use 20% for this ratio

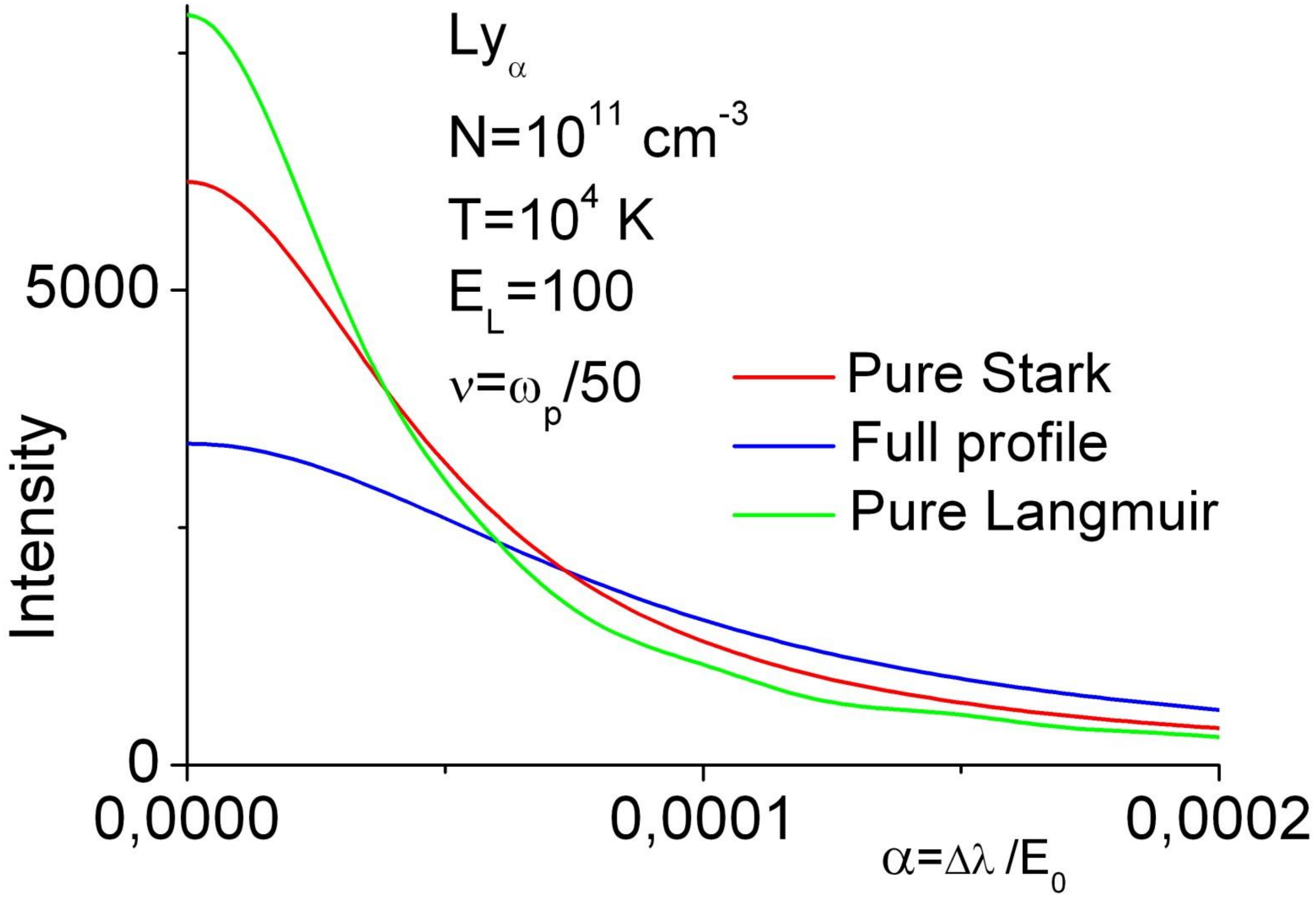
Ly_α line shape

We use α units $\alpha = \frac{\Delta\lambda}{E_0}, [\Delta\lambda] \equiv a\text{ngström}, [E_0] \equiv CGS,$ $(Griem)$



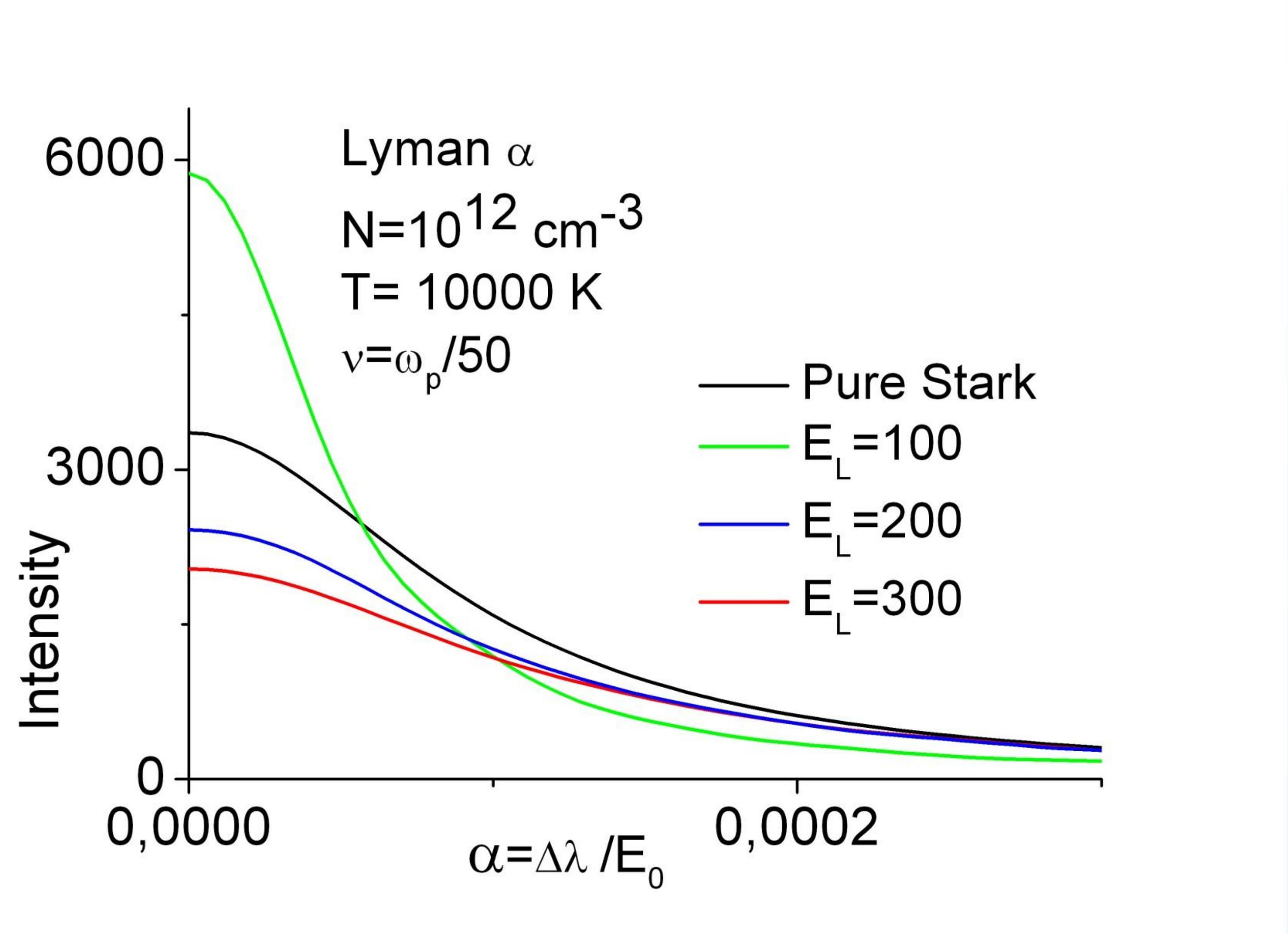
Full profile about 50% broader than the pure Stark profile

Ly_α line shape



Full profile twice broader than the pure Stark profile

Ly_α line shape: effect of Langmuir field amplitude



Saturation behaviour for E_L>200 E_0

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Conclusion

In presence of an intense energy source, coupling nonlinearly the plasma waves, strong Langmuir turbulence can develop in a plasma

Numerous wave packets appear and evolve in the plasma

The electric field peak values may be locally 2 to 3 orders of magnitude larger than the plasma microfield

A stochastic model has been proposed for calculating the effect of strong turbulence on a line shape

Our model predicts a strong additionnal broadening in the case of the hydrogen Ly_α line

Static and impact approximations

Two limiting regimes

Static:

$$\tau_i \ll \tau_c$$

$\tau_i \approx 1/\Delta\omega$ Time of interest

$\tau_c \approx r_0 / \mathrm{v}$ Collision time

$$\Rightarrow \quad \vec{E}(t) \approx \vec{\mathrm{E}}(0)$$

Only the PDF P(E) is needed

Impact:

$$\tau_i \gg \tau_c \quad \Rightarrow$$

Binary collisions
(electrons)