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ON THE MASS – LUMINOSITY RELATION

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Summary. The empirical mass–luminosity relation is analysed for the main sequence (MS) within the interval $-2.98 \leq L/L_{\odot} \leq 5.17$. The agreement with theoretical models is very satisfactory.

INTRODUCTION

For the components of various binary-system types for which the masses and the absolute bolometric magnitudes are known, one can construct an empirical M–L diagram. The first modern survey of the measured M and L (Harris *et al.*, 1963) comprises the visual binaries with $M{\sim}M_{\odot}$. The higher masses appear in the lists of Stothers (1972, 1973) concerning the components of eclipsing binaries and of Hutchings (1976) dealing with the case of spectroscopic ones. The survey of McClusky, Kondo (1972) for the visual and spectroscopic binaries also contains about one hundred systems in the case of which only the total mass is available. Finally, Popper's (1980) list contains the modern (accurate) determinations of the thermal characteristics, M, L and R, preferably for the stars of luminosity class V.

THEORY AND OBSERVATIONS

The dimension analysis for the homologous stars in the hydrostatical and thermal equilibrium yields the relation

$$L = CM^{q}, \tag{1}$$

with parameters C and q. Approximatively, for the laws of energy (ϵ) and opacity (κ) in the forms

$$\epsilon = \epsilon_0 \rho^{\lambda} T^{\nu}, \quad \kappa = \kappa_0 \rho^{n} T^{-s} \tag{2}$$

(ϵ_0 and κ_0 are known functions depending on the chemical composition only) the parameters C and q depend on the given X, Y, Z, λ, n and s, so that (1) is valid for radiative envelopes (in any case for the stars in radiative equilibrium). For the stars of the same (and homogeneous) chemical composition, with the same laws (2) and with the equation of state $P{\sim}\rho T$, C and q are constants — hence in that case (1) becomes

$$lg\frac{L}{L_{\odot}} = const + q\; lg\frac{M}{M_{\odot}} \tag{3}$$

with

$$q = \frac{3(3\lambda + n + \nu) + 2n\nu + s(2\lambda - 1)}{3(n+\lambda) + \nu - s}. \tag{4}$$

Fig. 1 gives an empirical lg$M$-lg$L$ diagram for the MS stars: 23 case of visual binaries, 108 case of eclipsing ones and 8 that of spectroscopic ones. It is seen that the theory yields a very good fit to the observations.

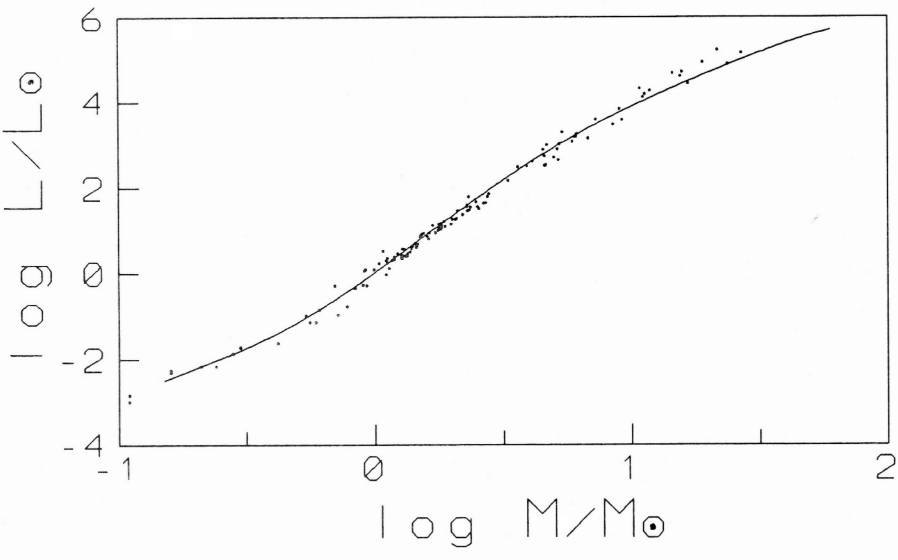

Fig. 1 Mass–luminosity diagram for the main sequence
... 139 stars from Popper's (1980) list,
— models with approximately solar chemical abundance
(from Tinsley, 1980).

Empirical relation (3) is given as a unique one for all stars in the chosen interval of L *i.e.* M along the main sequence: $q = 2.76$ for $-2.5 \leq \lg l \leq -1.1$ ($l = L/L_\odot$) and $q = 4$ for $-1.1 \leq \lg l \leq 1.9$ (Harris *et al.*, 1963); $q = 3.87$ for $-2.3 \leq \lg l \leq 5.1$ (McClusky, Kondo, 1972); $q = 3.01$ for $4.7 < \lg l < 6.7$ and $q = 2.7$ for the most massive stars only (de Jager, 1980). In Fig. 2a the rate of the luminosity change based on (3) is given for various L intervals, whereas in Fig. 2b one presents the same thing but for the cubic polynomial lg$L$(lg$M$) in the entire main sequence domain of Fig. 1.

As seen (Fig. 2a), for each of the main sequence intervals, the theoretical luminosity variations from the local approximation (3) exceed the observed ones. On the other hand, the general fit does not produce any systematic excess and it yields a very good agreement between the models and the observations in an extended surrounding of $M_\odot$ (Fig. 2b). In both cases (observations and models) the maximum values for q and $d\lg L/d\lg M$ correspond to the domain $M \sim M_\odot$. Formula (4) for Kramers opacity ($n = 1, s = 3.5$) and the pp reaction chain with $\nu = 2.5 - 4.5 (\lambda = 1)$ yields $q = 5.4 - 5.6$, and for the CN cycle with $\nu = 13 - 22$ one obtains $q = 5.1 - 5.2$. At the same time, approximation (3) about the maximum (Fig. 2a) yields $q = 3.7$ for the observations and $q = 4.3$ for the models (in Fig. 2b the maximum also occurs at 4.3). For Thomson scattering by free electrons ($n = 0, s = 0$) formula (4) yields

$q = 3$ independently of the values for λ and ν, though it is clear that the CN reactions appear (for hot - massive stars).

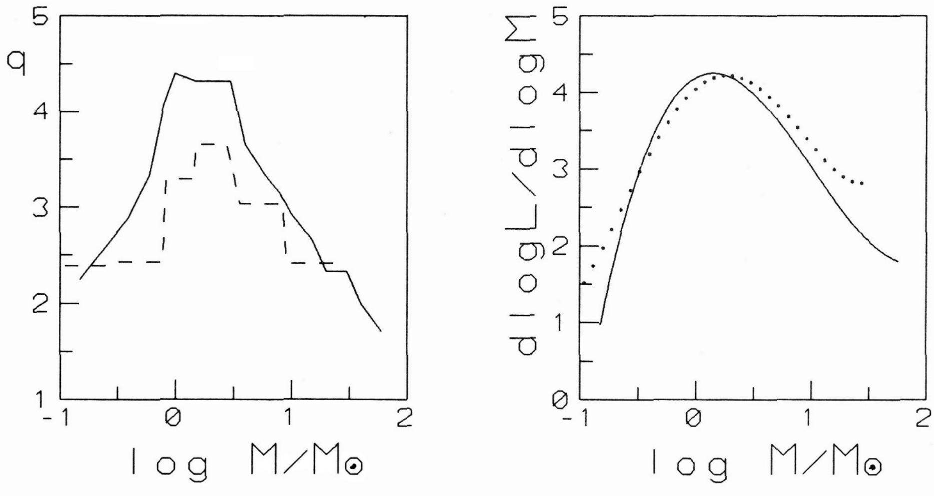

Fig. 2 Luminosity-change rate
left(a): from (3); right(b): from cubic $\lg L(\lg M)$;
- - - (...) observations; — models.

In the domain $M<M_{\odot}$ approximation (2) for κ is not valid in view of the dominant convection in the case of low-mass stars.

CONCLUSION

The theoretical stellar models for the main sequence phase yield very good fit to the observations on the mass–luminosity diagram. Here, on the basis of (local) linear relation (3), the models yield a systematically more rapid luminosity change with mass. In the case of the nonlinear approximation $\lg L(\lg M)$ the differences are significantly smaller (negligible for $M{\sim}M_{\odot}$) and not systematic. In both cases, q_{max} and $(dlgL/dlgM)_{max}$ are in the domain $M{\approx}2M_{\odot}$ where (according to theory) occurs the change in the transfer mechanism in the envelopes of the MS stars.

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