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THE ORIENTATION OF THE FK5 COORDINATE SYSTEM FROM THE MERIDIAN OBSERVATIONS OF THE SUN AND PLANETS

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Abstract: In this paper the elements of orientation of the fundamental system and the procedure for their calculation are presented. On the basis of the observation of the Sun, Mercury and Venus computed are the corrections ΔA and $\Delta\delta_o$.

1. Introduction

The determination of star coordinates and the compilation of the corresponding catalogues is not only important, but a central problem in astrometry today.

There is a constant interest in obtaining a knowledge of the structure of the Universe in space and time as good as possible. This requires the question of establishing of a fundamental coordinate system, necessary to determination of the star positions and proper motions of high accuracy, to be considered. Consequently the instruments, the measuring techniques and the calculation methods have been improved.

For the sake of compilations of new catalogues and improvements of the fundamental systems obtained earlier large observational programmes have been initiated, the instruments have been improved and automatised (photoelectric registration), new principles of formation of space coordinate systems have been extended and examined, the revolution and rotation of the Solar-System bodies have been studied, the composition of the terrestrial atmosphere has been examined, etc.

In this complex programme an important place is occupied by the tasks of studying the systematic catalogue errors, i.e. of determining the zero point for the star coordinates and the periodical errors of the catalogue system.

Whenever a new fundamental catalogue is compiled, appears the problem of determining an origin of the coordinate system as good and as accurate as possible.

Over the last 150 years the improvement of the coordinate system's origin has been done five times and hence some new ideas concerning the relationship with alternative, new, possibilities for determining the orientation of the fundamental coordinate system have appeared.

A more general approach to the question of the orientation of the coordinate axes in the star catalogues has been specially extended and also methods for solving these problems have been proposed. In order to improve the accuracy in the determining the position of the right-ascension zero point the list of observed objects has been enlarged, an increased number of modern measurements has been collected, etc.

All of this enables the task of the zero-point determination to be solved with a high accuracy.

The present examinations were undertaken after the recommendation at the XV IAU General Assembly and in 1984 it was decided to derive a new fundamental system comprising 3500 FK5 bright and faint stars and being based on the improvement of the FK4 fundamental system.

2. The Orientation of the Coordinate Axes

The determination of the coordinate-origin position (so-called zero point) and of the orientation angles for the coordinate axes in star catalogues consists of the calculation of the constant declination correction $\Delta\delta_o$ and of the orientation elements ΔA.

The equator of a catalogue KK is a small circle on the celestial sphere, parallel the great circle K_1K_1 and it is at the distance $\Delta\delta_o$ from the latter one (Fig.1). This distance is positive if KK is north of K_1K_1 and vice versa. The great circle TT is the dynamical or true equator. The true point of equinox N is defined as the intersection of the dynamical equator TT with the ecliptic EE. We shall denote as $x_ky_kz_k$, $x_1y_1z_1$ and xyz the coordinate systems corresponding to the circles on the sphere mentioned above. The axis of abscissae in the system xyz is directed from the centre of the sphere towards the point N_k, whereas the the ordinate perpendicular to it goes through the circle KK. In the rectangular coordinate system $x_1y_1z_1$ the abscissae axis is directed towards the point N_k' corresponding to the position of the catalogue equinox on the great circle K_1K_1 after the displacement of the catalogue equator KK by the distance $\Delta\delta_o$ (Fedorov, 1974).

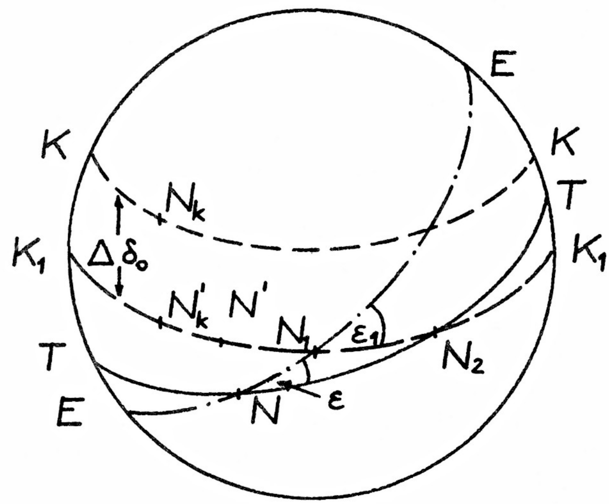

Fig. 1. Position of the equator of a catalogue with relation to the dynamical equator and the ecliptic

The ordinate axis of the coordinate system $x_1y_1z_1$ is in the plane of the great circle K_1K_1 . The axes z_k and z_1 are directed towards the pole of the catalogue equator. In the rectangular coordinate system xyz the axis x is directed towards the true-equinox point N, while the plane xy coincides with the equator one.

The question of axes orientation is reduced to the determination of the rotation angles for the rectangular coordinate system whose x_1y_1 plane coincides with the plane of the corrected equator K_1K_1 with respect to the axes of the rectangular coordinate system attached to the true equator TT.

The rotation angles are easily obtained by transforming the catalogue positions α_k, δ_k into the coordinates α, δ determining the planet positions in the true coordinate system. This transition is done in two phases: the catalogue equator KK is translated parallelly to itself to the distance $\Delta\delta_o$ so that it obtains the position K_1K_1 and the point N_k falls to the position N'_k . Then the system $x_1y_1z_1$ should be rotated to coincide with xyz .

This transformation may be also done in the opposite sense. The first rotation is done for the system xyz around the axis z by an angle $u = NN_2$, then by $\Delta\varepsilon$ equal to the angle N'_kN_2N around the new position of the axis x_1 going through the point N_2. The last rotation concerns the system by an angle $\omega = N'_2N'_k$ around the new position of the axis z.

When the circle TT is rotated by the angle $\Delta\varepsilon$, the point N occupies the position N' and $\omega - \alpha = N'_kN_2 - NN_2 = N'_kN_2 - N'N_2 = \Delta\alpha_o$, where $\Delta\alpha_o$ is the difference in the positions of the catalogue equinox and the true one on the equator K_1K_1 . Let the quantity $\Delta A = -\Delta\alpha_o$ be a constant correction of the catalogue right ascension for the conversion to the equinox N. Its practical role is the improvement of the star position in a catalogue. The name used in the literature for ΔA is "the equinox correction".

In order to determine the corrections of the coordinate-system origin (so- called zero point) and the orientation angles of the coordinate axes in star catalogues one should calculate the declination-correction constants $\Delta\delta_o$ and the orientation elements ΔA, $\lambda_1 - \lambda = \Delta\lambda$, $\varepsilon_1 - \varepsilon = \Delta\varepsilon$ throught which the rotation angles of the axes u, $\Delta\varepsilon$, ω are given.

The differences between the catalogue coordinates and the true ones in right ascension and declination are determined by means of the following formulae

$$\alpha_k - \alpha = -\Delta A \quad , \quad \delta_k - \delta = -\Delta\delta_o \tag{1}$$

where the displacement of the catalogue equator with respect to the dynamical one is not present.

For the purpose of determining the coordinate-system origin one utilises the differences $\alpha_k - \alpha_e$, $\delta_k - \delta_e$ where α_k and δ_k denote the corresponding right ascensions and declinations of the planets obtained from observations by applying the differential method in the reference-catalogues system, whereas α_e and δ_e are their ephemeris right ascensions and declinations

$$\alpha_k - \alpha_e = (\alpha_k - \alpha) + (\alpha - \alpha_e) \qquad \delta_k - \delta_e = (\delta_k - \delta) + (\delta - \delta_e). \tag{2}$$

The differences $\alpha_k - \alpha$ and $\delta_k - \delta$ are the changes of the coordinates due to the rotation of the dynamical equator with respect to the catalogue one, whereas the ones $\alpha - \alpha_e$ and $\delta - \delta_e$ are due to the errors in the determination of the orbits (Earth and other planets) used in the ephemeris calculation.

The differences between the catalogue right ascensions, i.e. declinations, and the true ones calculated by use of the formulae

$$[\alpha_k - \alpha, \delta_k - \delta] = [\Delta A, \Delta\delta_o, (\lambda_1 - \lambda)\sin\varepsilon, \varepsilon_1 - \varepsilon]M_1 \tag{3}$$

where is M_1 is the transformation matrix and (2) yield

$$[\alpha_k - \alpha_e, \delta_k - \delta_e] = [\Delta M_o + \Delta r, \Delta p, \Delta q, e\Delta r, 100\Delta a/a, \Delta e,$$
$$\Delta M_o' + \Delta\psi_3', \Delta\psi_1', \Delta\psi_2', 10e'\Delta\psi_3', 100\Delta e']M_2M_3 +$$
$$+[\Delta\delta_o, \Delta\lambda \sin\varepsilon, \Delta\varepsilon_1]M_4 \quad . \tag{4}$$

ΔM_o, Δe, $\Delta M_o'$, $\Delta e'$ denote the corrections of the mean anomaly and eccentricity of the planet orbits (a dash for Earth, without it for other planets), whereas Δa denotes the correction of the semimajor axis. The components of the vector of planet-orbit rotation in the rotating rectangular coordinate system Δp, Δq, Δr are related to the corrections of the orbital-node longitude $\Delta\Omega$, inclination Δi and angular distance node-perihelion $\Delta\omega$ by means of

$$\Delta i = \Delta p \cos\omega - \Delta q \sin\omega \ , \quad \Delta\Omega \sin i = \Delta p \sin\omega + \Delta q \cos\omega \ , \quad \Delta\omega + \Delta\Omega \cos i = \Delta r \ . \tag{5}$$

The components of the Earth-orbit-rotation vector in the equatorial rectangular coordinate system $\Delta\psi_1'$, $\Delta\psi_2'$, $\Delta\psi_3'$ appear through the corrections of the inclination $\Delta\varepsilon'$ at the angular distance between the perihelion and the node of the Earth's orbit $\Delta\omega'$ and the difference between the catalogue equinox and the dynamical one ΔA in the mutual relationship (Duma and Minyajlo, 1976) :

$$\Delta\psi_1' = \Delta\varepsilon' \ , \quad \Delta\psi_2' = -\Delta A \sin\varepsilon \ , \quad \Delta\psi_3' = -\Delta A \cos\varepsilon + \Delta\omega' \ . \tag{6}$$

Equations (4) contain 14 unknown quantities: the six corrections of the orbital elements (ΔM_o, Δi, $\Delta\Omega$, $\Delta\omega$, Δa, Δe), the four corrections of the Earth's orbital elements ($\Delta M_o'$, $\Delta\varepsilon'$, $\Delta\omega'$, $\Delta e'$), the three orientation elements (ΔA, $\Delta\lambda$, $\Delta\varepsilon_1$) and $\Delta\delta_o$ - the element of the sharp angle of the catalogue coordinate system.

Namely, equation (3) in the treatment of the planet observations is used for the purpose of correcting the zero point position in the coordinate system of the fundamental catalogue FK5.

In favour of this statement are also the values of the right ascension and declination corrections for the FK5 stars obtained on the basis of the Belgrade observational material for the period 1975 - 1991:

$$\Delta A = 0^s\!.014 \pm 0^s\!.006$$
$$\Delta\delta_o = 0''\!.05 \pm 0''\!.05 \quad .$$

3. Conclusions

On the basis of all said above one can say that the Belgrade observations of the Sun and planets with regard to both systematic and random errors are not inferior to the observations carried out at Washington, Greenwich, Cape, Nikolaev, Pulkovo, etc, but they even appear very good in quality when compared to them. Finally, we think that the number of observations should be made as large as possible, the behaviour of the instrument should be rigorously controlled and during the reduction procedure all possible influences should be taken into account in order to make them less significant.

References

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