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NONLINEAR COHERENT STRUCTURES IN PLASMAS AND FLUIDS

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Abstract. Nonlinear electrodynamic and hydrodynamic equations having solutions in the form of coherent stationary vortices and vortex chains are presented. In most cases they are capable of depicting the observed nonlinear structures in various laboratory and space plasmas.

Nonlinear coherent structures, double and monopole vortices (Hasegawa and Mima, 1977), and vortex chains (Shukla and Stenflo, 1995; Vranješ and Jovanović, 1997), resulting from the self-organization of both fusion and space plasmas, have attracted a lot of interest in the last twenty years. They may appear in various processes such as nonlinear interaction of a strong pump propagating through a plasma in the processes of plasma heating, with slow low frequency perturbations normally existing in plasmas (Jovanović and Vranješ, 1996), in the development of some plasma instabilities (Aleksić *et al.* 1996; Jovanović and Vranjeǐ990, 1994) etc. Recently it has been shown that equations describing low frequency electrostatic waves in a sheared plasma flow in the auroral ionosphere (Shukla *et al.* 1995) and waves in large self-gravitating astrophysical clouds (Shukla and Stenflo, 1995), possess solutions in the form of traveling vortex chains periodic in the direction of propagation and localized transversely to it. Recent satellite observations of electromagnetic structures in the Earth's ionosphere and magnetosphere, and corresponding theoretical model equations (Chmyrev *et al.* 1991), reveal that apart from monopole and dipole structures, there also may exist coherent solutions in the form of vortex chains. It is believed that the manifestation of such vortical structures in the magnetosphere are the discrete fluxes of electrons in active auroral forms, which occur due to acceleration of electrons by the electric field component directed along the magnetic field lines. Vortex solitons obtained recently from the Freja satellite (Wu *et al.* 1997) with characteristic spatial scales of 300-600 m, can be nicely described using standard nonlinear theory of drift waves, developed in order to describe the drift wave turbulence in present day tokamak machines.

A typical equation describing vortices may be derived using the standard model of two-component, electron-ion plasma immersed in an external magnetic field $B_0\vec{e}_z$, with the equilibrium density gradient dn_0/dx, and in the limit of low-frequency (in comparison with the ion gyrofrequency) electrostatic perturbations. Using standard hydrodynamic equations, i.e. the ion continuity and momentum equations, and the assumption of quasineutrality and Boltzmann distribution of electrons, one can obtain the well known Hasegawa-Mima equation (Hasegawa and Mima, 1977) which can be written in the form:

$$\frac{\partial}{\partial t}\left(\rho^2\nabla^2\Phi-\Phi\right)-\frac{\rho^2}{B_0}\left[(\nabla\Phi\times\vec{e}_z)\cdot\nabla\right]\left[\nabla^2\Phi-\frac{T_e}{\rho^2 e}\log\left(\frac{n_0}{\Omega_i}\right)\right]=0. \qquad (1)$$

Here the assumption of cold ions is used and ρ, Ω_i, T_e, Φ are the ion gyroradius, ion gyrofrequency, electron temperature, and electrostatic potential, respectively. In the linear regime Eq. (1) describe ordinary drift waves. The most simple nonlinear process described by the above equation is the three wave interaction in plasmas. A characteristic cascading of wave energy towards larger and smaller wave numbers k can be easily demonstrated, meaning that the mode with the intermediate value of k may act as a pump. In the strongly nonlinear regime Eq. (1) possesses solution in the form of a double vortex traveling with a constant velocity perpendicularly both to the density gradient, and the magnetic field lines. Although it is not the soliton in the strict sense it is remarkably stable and may survive different types of perturbations like collisions with other vortices etc.

It is interesting to note that there exists an analogous equation in the hydrodynamic theory of Rossby vortices; exactly the same equation describing cyclones and anticyclones in the Earth's atmosphere may be obtained replacing Φ by the perturbation of the surface of the atmosphere δh, and the ion gyroradius by the Rossby-Obukhov radius r_R. The best known example of the monopole solution of the Rossby equation is the Great Red Spot on Jupiter, observed by R. Hooke as far back as in 1664, and described in his paper (Hooke, 1666). It is an anticyclone vortex which extends about 26 000 km in longitude and 13 000 km in latitude, drifting westward with the velocity of about 3 m/s, and can be modeled by relatively simple experiments with rotating fluids (Nezlin, 1986). The White Ovals, lasting for more than 50 years, and Brown Ovals (more than 10 years) are another examples of long lasting vortices on Jupiter.

Vortex-type structures may also be found in large self-gravitating magnetized plasma clouds (Jovanović and Vranješ, 1990). Starting from a model of a homogeneous cold plasma cloud we study Alfven-type two dimensional perturbations propagating perpendicularly to the magnetic field. Using standard electro-hydrodynamic equations with the gravitational effects included via Poisson equation one can obtain the following set of coupled nonlinear equations describing perturbations of electrostatic (Φ) and gravitational (Γ) potential:

$$\left[\frac{\partial}{\partial t}+\frac{1}{B_0}(\vec{e}_z\times\nabla_\perp\Phi)\cdot\nabla_\perp\right]\left[\frac{\Omega_i^2}{\omega_{pi}^2}\nabla_\perp\Phi-(\nabla_\perp^2+1)\Gamma\right]=0, \qquad (2)$$

$$\left\{\frac{\partial}{\partial t}+\frac{1}{B_0}\left[\vec{e}_z\times\nabla_\perp\left(\Phi-\frac{\Omega_g^2}{\Omega_i^2}\Gamma\right)\right]\cdot\nabla_\perp\right\}\left[\nabla_\perp^2\Phi+(\nabla_\perp^2+1)\Gamma\right]=0. \qquad (3)$$

Here the magnetic field is in the z-direction, $\nabla_\perp=\partial_x\vec{e}_x+\partial_y\vec{e}_y$, $\omega_g^2=4\pi G m_i n_0$, $\omega_{pi}^2=e^2 n_0/m_i\epsilon_0$, and m_i is the ion mass. Nonlinear Eqs. (2), (3) have solutions in the form of double vortex with typical scale size of the order of Jeans' critical length.

It can be shown also that the gravitational collapse is a higher order effect, i.e. it occurs on a much longer time scale compared to the characteristic time (e.g. period of rotation) of the vortex.

Three dimensional nonlinear Rossby waves in rotating gravitating systems with a nonuniform angular velocity $\omega_0(x,y)\vec{e}_z$, and with a perpendicular equilibrium density gradient have been studied in Ref. Shukla and Stenflo, 1995). Using standard fluid equations one can obtain the following equation describing the perturbation of gravitational potential Γ:

$$\begin{gathered}\frac{\partial^2}{\partial t^2}\nabla_\perp^2\Gamma + \frac{1}{2\omega_0}\frac{\partial}{\partial t}J(\Gamma,\nabla_\perp^2\Gamma) + \frac{1}{\alpha-2}\left(\frac{\partial}{\partial t}\nabla_\perp\Gamma\times\vec{e}_z\right)\nabla_\perp\left(\frac{\omega_g^2}{\omega_0}\right) \\ + \frac{2}{\alpha-2}\left(\omega_g^2 - \frac{\partial^2}{\partial t^2}\right)\frac{\partial^2\Gamma}{\partial z^2} = 0.\end{gathered} \tag{4}$$

Here $\alpha = 2\pi G n_0/\omega_0^2 = \omega_g^2/2\omega_0^2$, $J(f,g) = \partial_x f\partial_y g - \partial_x g\partial_y f$. In the linear limit the above equation describe linear Rossby waves. In the strongly nonlinear case an analytical stationary solution of Eq. (4), traveling with a constant velocity u in the y-direction, for u satisfying certain conditions, can be written in the form:

$$\Gamma = 2u\omega_0 x + A\log 2\left[\cosh(kx) + \sqrt{1-1/a^2}\cos(ky)\right]. \tag{5}$$

For $a^2 > 1$ it represents a vortex street resembling the Kelvin-Stuart cat's eyes.

Similar solutions may be found numerically in the problem of self-generation of magnetic field (Vranješ and Jovanović, 1996). We study electromagnetic perturbations of electrons in an electron-ion plasma with heavy ions making neutralizing background, and with an electron flow in the basic state. Using the electron momentum and energy equation together with the Maxwell equations one can find the following nonlinear equation describing the generation of magnetic field:

$$\left(\frac{\partial}{\partial t} + \varphi'\frac{\partial}{\partial y} + \vec{e}_z\times\nabla B\cdot\nabla\right)(\nabla^2-1)B - \varphi'''\frac{\partial B}{\partial y} = 0. \tag{6}$$

This equation is derived on condition of a weak time dependence $\partial_t \ll \omega_{pe}$ (electron plasma frequency), for z-independent perturbations, and for the plasma flow given by $\vec{v}_0(x) = V_0 f(x)\vec{e}_y$, where we introduced $\varphi'(x) = f(x)$. In the linear case Eq. (6) belongs to the class of equations describing streaming instabilities. In the strongly nonlinear limit we look for stationary solutions traveling along the y-axis with the velocity u. Writing $\partial/\partial t = -u\partial/\partial y$, Eq. (6) can be integrated once, and for the flow profile symmetric around the phase velocity u it is solved numerically. The solutions are sought in the form:

$$B(x,y) = B_1(x) + \delta B_1(x)\cos(ky), \quad |\delta B_1(x)| << |B_1(x)|. \tag{7}$$

We found two different nonlinear modes, for even and odd B_1 and δB_1. In the first case the solution for the magnetic field is a two-dimensional, single vortex chain structure, localized along the gradient of the flow (x-axis), and periodic in the perpendicular direction (y-axis). In the second case it has the form of a double chain structure with much bigger periodicity length (smaller k_y) along the y-axis.

In both cases the self-generated magnetic filed yields a significant steepening of the electron flow profile. A similar situation was observed in the case of a magnetized plasma (Vranješ, 1998). In a local approach perpendicularly to the flow, double vortices driven by the density and temperature gradients are found. In a nonlocal case we obtain vortex chains driven by the flow.

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