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MAGNETIC FIELD GENERATION AT ION ACOUSTIC TIME SCALE

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Abstract. Ion acoustic wave in an inhomogeneous plasma naturally couples with a transverse electromagnetic perturbations. Due to this coupling the ion acoustic mode becomes electromagnetic. There appears a lower frequency cut off of the ion acoustic wave, the wave becomes dispersive and backward.

1. INTRODUCTION

The model of electron-magnetohydrodynamics (EMHD) includes electron perturbations on a background of immobile ions. Usually, the displacement current in the Ampère law is neglected in this model, implying $(\omega/k)^2 \ll c^2$, where ω and k denote the wave parameters, and c denotes the speed of light. Moreover, in a plasma containing only electrons and static ions, the perturbations are usually assumed to be incompressible. This is formally seen by substituting the Ampère's law (without the displacement current $\mu_0\varepsilon_0\partial\vec{E}/\partial t$) into the electron continuity equation. The mentioned assumption of static ions implies electron perturbations that take place at spatial scales at which the ions, due to their much larger inertia, are not able to react, i.e., much below the ion skin depth $\lambda_i \equiv c/\omega_{pi}$, where ω_{pi} is the ion plasma frequency. This may formally be checked by making the ratio $|n_i\vec{v}_i|/|n_e\vec{v}_e|$, and by further using the ion momentum equation and the Ampère law to express the ion and electron velocities, and by applying the operator $\nabla\times$ onto the resulting expression in the numerator and denominator separately. This yields

$$\left|\frac{\omega_{pi}^2}{\omega c^2}\frac{\nabla\times\vec{E}}{\nabla\times(\nabla\times\vec{B})}\right| = \frac{1}{\lambda_i^2 k^2} \ll 1.$$

In the presence of an equilibrium density gradient, using the EMHD set of equations together with an electron energy (temperature) equation, such perturbations describe the generation of magnetic field in plasmas with sharp discontinuities (the magnetic surface waves) Jones (1983), Yu and Stenflo (1985), or in weakly inhomogeneous plasmas Nycander et al. (1987). The magnetic field is generated due to the baroclinic vector $\nabla n_0 \times \nabla T_{e1}$ (where T_{e1} is the electron temperature perturbation). This mechanism was first suggested by Stamper and Tidman (1973), and is essentially an electron dynamics effect.

In the present work we give an alternative approach to the problem of the generation of magnetic field in plasmas. We shall show that when the ion perturbations are taken into account (implying compressible perturbations), there appears a coupling between the ion acoustic and electromagnetic perturbations. The coupling is entirely due to the presence of small equilibrium density and temperature gradients. Note that here both equilibrium gradients enter into the mode description only in order to have a properly satisfied equilibrium. This means that the two gradient vectors are antiparallel.

2. DERIVATIONS AND RESULTS

To demonstrate the coupling we use a simple model with a minimum number of equations forming a closed set. We start from the wave equation for the perturbed EM field:

$$k^2\vec{E}_1 - \vec{k}(\vec{k}\cdot\vec{E}_1) - \frac{\omega^2}{c^2}\vec{E}_1 - \frac{i\omega}{c^2\varepsilon_0}\vec{j} = 0. \tag{1}$$

Here, the perturbations are assumed to be of the form $\sim f(x)\exp(-i\omega t + ikz)$. In addition, we assume that the equilibrium plasma has small density and temperature gradients along the x-axis. Therefore, a weak x-dependence of the wave amplitude is considered along with the local approximation implying $|\partial/\partial x| \ll |k| = 2\pi/|\lambda_z|$. The perturbations are assumed to be isothermal (contrary to the usual EMHD theory) and the ions are assumed to be cold. Having the density and temperature gradients along the x-axis, we choose perturbations propagating in any perpendicular, e.g., z-direction. The properties of the medium enter Eq. (1) through the current $\vec{j} = en_0(x)(\vec{v}_{i1} - \vec{v}_{e1})$.

The electron dynamics must include the momentum and continuity equations while the ions are completely described by $m_i n_0 \partial\vec{v}_{i1}/\partial t = en_0\vec{E}_1$. A static plasma equilibrium without macroscopic electromagnetic field is satisfied by

$$\frac{1}{T_0}\frac{dT_0}{dx} = -\frac{1}{n_0}\frac{dn_0}{dx}. \tag{2}$$

The electron velocity is given by

$$\vec{v}_{e1} = -\frac{ie}{m_e\omega}\vec{E}_1 + \frac{iev_{Te}^2}{\omega m_e\omega_0^2}\nabla(\nabla\cdot\vec{E}_1) + \frac{iev_{Te}^2}{\omega m_e\omega_0^4}(\nabla\cdot\vec{E}_1)k^2v_{Te}^2\frac{\nabla T_0}{T_0}$$
$$+\frac{iev_{Te}^2}{\omega m_e\omega_0^2}\frac{\nabla(E_{1x}n_0')}{n_0} + \frac{iev_{Te}^2}{\omega m_e\omega_0^2}\frac{E_{1x}n_0'}{n_0}\frac{k^2v_{Te}^2}{\omega_0^2}\frac{\nabla T_0}{T_0} + \frac{iev_{Te}^2}{\omega m_e\omega_0^2}\frac{E_{1x}n_0'}{n_0}\frac{\nabla T_0}{T_0}. \tag{3}$$

Here, $\omega_0^2 = \omega^2 - k^2v_{Te}^2$, $v_{Te}^2 = \kappa T_0(x)/m_e$, $\nabla n_0 = \vec{e}_x dn_0/dx \equiv \vec{e}_x n_0'(x)$, $\nabla T_0 = -\vec{e}_x dT_0/dx \equiv -\vec{e}_x T_0'(x)$. We have assumed a linear x-dependence of the two equilibrium quantities $n_0(x)$ and $T_0(x)$. Eq. (1) becomes

$$k^2\vec{E}_1 - \vec{k}(\vec{k}\cdot\vec{E}_1) - \frac{\omega^2}{c^2}\vec{E}_1 + \frac{\omega_{pe}^2+\omega_{pi}^2}{c^2}\vec{E}_1 + \frac{\omega_{pe}^2 v_{Te}^2}{c^2\omega_0^2}\vec{k}(\vec{k}\cdot\vec{E}_1)$$
$$-\frac{i\omega_{pe}^2 v_{Te}^2}{c^2 n_0\omega_0^2}\,\vec{k}E_{1x}n_0' - \frac{ik^3 v_{Te}^4\omega_{pe}^2}{c^2\omega_0^4 T_0}E_{1z}T_0'\vec{e}_x = 0. \qquad (4)$$

The y-component of the vector equation (4) gives the transverse electromagnetic mode which is not of interest here.

The z and x components are

$$\left(-\omega^2 + \omega_{pe}^2 + \omega_{pi}^2 + \frac{k^2\omega_{pe}^2 v_{Te}^2}{\omega_0^2}\right)E_{1z} - \frac{ikv_{Te}^2\omega_{pe}^2 n_0'}{\omega_0^2 n_0}E_{1x} = 0, \qquad (5)$$

$$\left(k^2 + \frac{\omega_{pe}^2+\omega_{pi}^2-\omega^2}{c^2}\right)E_{1x} - \frac{ik^3 v_{Te}^4\omega_{pe}^2 T_0'}{c^2\omega_0^4 T_0}E_{1z} = 0. \qquad (6)$$

Eqs. (5) and (6) describe the coupling (due to the small density/temperature gradients) between the longitudinal, E_{z1}, and the transverse electric field perturbation, E_{x1}. They also determine the transverse, perturbed magnetic field component $B_{y1} = kE_{x1}/(i\omega)$. The dispersion equation reads:

$$\left(\omega_1^2 - \frac{k^2 v_{Te}^2}{\omega_{pe}^2}\right)^2\left[\omega_1^2\left(\frac{\omega_{pi}^2}{\omega_{pe}^2} + 1 - \omega_1^2\right) - \frac{k^2 v_{Te}^2}{\omega_{pe}^2}\left(\frac{\omega_{pi}^2}{\omega_{pe}^2} - \omega_1^2\right)\right]\left(1 + \frac{k^2c^2}{\omega_{pe}^2} + \frac{\omega_{pi}^2}{\omega_{pe}^2} - \omega_1^2\right)$$
$$+\frac{k^4 v_{Te}^6}{\omega_{pe}^6}\frac{n_0'}{n_0}\frac{T_0'}{T_0} = 0. \qquad (7)$$

Here, $\omega_1 \equiv \omega/\omega_{pe}$. Our interest here is the low frequency limit $\omega/\omega_{pe} \ll 1$, which yields the mode frequency in the range of the ion sound wave:

$$\omega_{EMIA}^2 = \frac{k^2c_s^2}{1+k^2\lambda_d^2}\left[1 + \frac{m_i}{m_e}\left(\frac{n_0'}{n_0}\right)^2\frac{1}{k^2(1+k^2\lambda_e^2)}\right]. \qquad (8)$$

In the limit of small values of the wave number, there appears a lower frequency cut off for the electromagnetic ion acoustic (EMIA) wave $\omega \to \omega_c = \kappa_n^2 v_{Te}^2$, where $\kappa_n = n_0'/n_0$. The group velocity of the electromagnetic ion acoustic mode (8) appears proportional to

$$k^4\lambda_e^4 - 2k^2\lambda_e^2(\kappa_n^2\lambda_d^2 m_i/m_e - 1) + 1 - \kappa_n^2 m_i(\lambda_e^2+\lambda_d^2)/m_e.$$

The sign of this group velocity is thus determined by the wave-number, the equilibrium density scale length, and the plasma parameters, so that we can have both a direct and a backward wave. As seen from Eq. (8), for the wave-lengths much exceeding the electron Debye radius, the mode may have a frequency which is above the ion sound frequency. On the other hand, the second term in Eq. (8) describes a backward mode. As a demonstration only, these properties are presented in Fig. 1 (Vranjes et al. 2007) for the ion mass $m_i = 40$ proton masses, for the electron temperature $T = 10^4$ K, and taking $\kappa_n = 1$ m^{-1}. We plot the EMIA mode for two values of the number density, viz. $n_0 = 10^{17}$ m^{-3} (dashed line) and $n_0 = 10^{15}$ m^{-3} (full line).

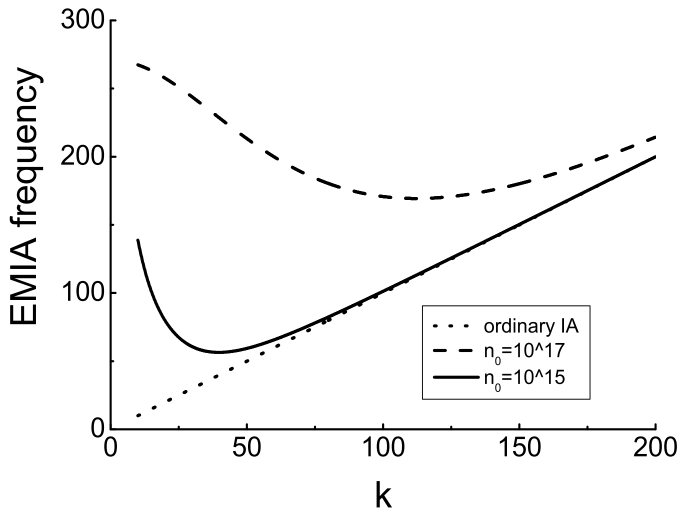

Figure 1: The electromagnetic ion sound frequency in units of $\kappa_n c_s$ for the two number densities $n_0 = 10^{17}$ m^{-3} (dashed line), and $n_0 = 10^{15}$ m^{-3} (full line) in terms of k normalized to κ_n.

The frequency is normalized to $\kappa_n c_s$ and the wave-number to κ_n. The dotted line describes the ordinary ion sound normalized to the same units. From the full and dashed lines it is seen that for small wave numbers it is a backward mode and very much different from the ordinary ion sound wave.

The linear dispersion equation Eq. (8) describes an electromagnetic wave in unmagnetized plasmas at the ion acoustic time scale, with the electromagnetic part which is due to the electron pressure gradient. However, for the electromagnetic part of the mode we do not need the electron temperature perturbation.

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