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# RELATIVISTIC COMPRESSION OF A LASER PULSE REFLECTED FROM A MOVING PLASMA

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**Abstract.** The reflection of an obliquely incident electromagnetic pulse from a moving plasma half-space is studied. Using the Lorentz transformations, covariance of Maxwell's equations and principle of phase invariance to transform between the rest frame and the moving frame, analytical formula for the linear reflected waveform shows temporal compression and pulse amplification at relativistic velocities of interest for generation of ultra-short laser optical pulses.

## 1. INTRODUCTION

A transient reflection and transmission of an obliquely incident EM pulse at the steady (non-moving) plasma-vacuum interface has been solved analytically in a closed form by Chabris and Bolle 1971, and Stanić and Škorić 1973ab. Recently, generation of ultra-short (attosecond range - $10^{-18}$s) light and relativistic particle bunches gained importance in various applications (see Mourou et al. 2006). Here, we revisit a general problem of a linear reflection of a time-dependent EM (laser) pulse from a plasma half-space moving at the relativistic velocity, see, Stanić and Škorić 1974.

### 1. 1. FORMULATION

A time-dependent electromagnetic plane wave pulse is incident at the moving cold plasma-vacuum interface. The incident angle is $\theta_i$ and the plane of incidence is $Oxz$, as shown in figure 1. The incident electric field (S- polarization) of the EM pulse in the time domain, by inverse Fourier transformation, is

$$\begin{aligned} E_{yi} &= (1/2\pi) \int_{-\infty}^{+\infty} E_0 \exp\left[j\left(\omega_i t - \mathbf{k}_i \mathbf{r}\right)\right] d\omega_i \\ &\equiv E_0 \delta\left(t - (x/c)\sin\theta_i + (z/c)\cos\theta_i\right), \end{aligned} \tag{1}$$

where $\delta(t)$ is the Dirac's function and $\omega_i$ and $\mathbf{k}_i$ are the angular frequency and the wavenumber vector in the observer's rest frame $K$, respectively. The uniform plasma half-space is moving with the velocity $\mathbf{v}$, where in two special cases: a) $\mathbf{v} = \mathbf{e}_x v$ and b) $\mathbf{v} = \mathbf{e}_z v$. The rest frame of the moving plasma is $K'$.

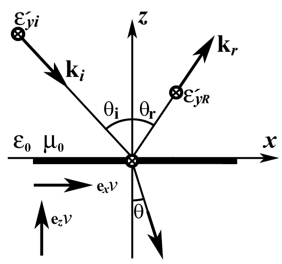


Figure 1: Geometry of the problem.

## 2. ANALYTICAL THEORY

Making use of the Lorentz transformations, covariance of Maxwell's equations and the principle of phase invariance, to transform between the rest (laboratory) frame and the moving frame (see e.g. Papas 1965, Ch. 7); the incident electric field in the moving frame $K'$, can be represented as

$$\begin{aligned} E'_{yi} &= (1/2\pi) \int_{-\infty}^{+\infty} \gamma \left(1 - \mathbf{k}_i \mathbf{v}/\omega_i\right) E_0 \exp\left[j\left(\omega_i t - \mathbf{k}_i \mathbf{r}\right)\right] d\omega_i = \\ &= (1/2\pi) \int_{-\infty}^{+\infty} \gamma \left(1 + \mathbf{k}'_i \mathbf{v}/\omega'_i\right) E'_0 \exp\left[j\left(\omega'_i t' - \mathbf{k}'_i \mathbf{r}'\right)\right] d\omega'_i, \end{aligned} \tag{2}$$

with physical quantites with the "prime" superscript corresponding to the moving frame $K'$, and where

$$\omega'_i = \gamma \left(1 - \mathbf{k}_i \mathbf{v}/\omega_i\right) \omega_i, \quad \gamma = \left(1 - v^2/c^2\right)^{-1/2} = \left(1 - \beta^2\right)^{-1/2}, \tag{3}$$

$$\mathbf{k}'_i = \mathbf{k}_i - \gamma \omega_i \mathbf{v}/c^2 + (\gamma - 1) \left(\mathbf{k}_i \mathbf{v}\right) \mathbf{v}/v^2, \quad \text{and} \tag{4}$$

$$E'_0 = \gamma \left(1 - \mathbf{k}_i \mathbf{v}/\omega_i\right) E_0, \quad E'_{01} = \gamma \left(1 + \mathbf{k}'_i \mathbf{v}/\omega'_i\right) E'_0. \tag{5}$$

With the $\exp\left(j\omega'_i t'\right)$ time dependence suppressed, the incident electric field in the frequency domain in the moving frame is given by

$$\mathcal{E}'_{yi} = E'_{01} \exp\left(-j\mathbf{k}'_i \mathbf{r}'\right), \tag{6}$$

and the frequency domain expression for the reflected field is simply

$$\mathcal{E}'_{yR} = \frac{1 - N'}{1 + N'} E'_{01} \exp\left(-j\mathbf{k}'_r \mathbf{r}'\right), \tag{7}$$

where, the well-known index of refraction for cold plasma at rest in $K'$, is

$$N' = \left|1 - \left(\omega'_p/\omega' \cos\theta'_i\right)^2\right|^{1/2}, \quad \omega'_i = \omega'_r = \omega'_t \equiv \omega'.$$

The vacuum dispersion relation $\omega_{i,r}^{(\prime)} = k_{i,r}^{(\prime)} c$, is valid in $K$ and $K'$ frame.

Using again the Lorentz transformations, covariance of Maxwell's equations and the principle of phase invariance to transform back from the moving frame $K'$ to the laboratory frame $K$, the time domain reflected field becomes

$$\begin{aligned} E_{yR} &= (1/2\pi) \int_{-\infty}^{+\infty} \gamma \left(1 + \mathbf{k}_r' \mathbf{v}/\omega_i'\right) \mathcal{E}_{yR}' \exp\left(j\omega' t'\right) d\omega' \\ &= \gamma \left(1 + \mathbf{k}_r' \mathbf{v}/\omega_i'\right) E_{yR}'. \end{aligned} \tag{8}$$

The expression for $E_{yR}'$ found by the standard method of contour integration as

$$E_{yR}' = -\left(2E_0'/\tau'\right) J_2\left(a'\tau'\right) U\left(\tau'\right), \tag{9}$$

where

$$\tau' = t' - \mathbf{k}_r' \mathbf{r}'/\omega', \quad a' = \omega_p'/\cos\theta_i', \quad \text{and } U\left(\tau'\right), \tag{10}$$

is the Heaviside unit step function, while $J_2(x)$ is the Bessel's function of the first kind of second order. We note that (9) is the Green's function solution, while a linear solution to another incident pulse profile is found by a convolution integration.

Further, we discuss two cases of the moving plasma half-space:

- a) $\mathbf{v} = \mathbf{e}_x v$

The reflected field is identical to non-moving plasma case; as normally incident wave does not "see" plasma motion in $x$-direction.

- b) $\mathbf{v} = \mathbf{e}_z v$

$$E_{yR} = -\left(2E_0\alpha_0/\xi\right) J_2\left(\alpha_1\xi\right) U\left(\xi\right), \tag{11}$$

where

$$\alpha_0 = \gamma^2\left(1 + 2\beta\cos\theta_i + \beta^2\right), \quad \alpha_1 = \left|\omega_p/\gamma\left(\beta + \cos\theta_i\right)\right|, \tag{12}$$

and

$$\xi = \alpha_0 t - (x/c)\sin\theta_i - (z/c)\gamma^2\left|\left(1+\beta^2\right)\cos\theta_i + 2\beta\right|. \tag{13}$$

## 3. RESULTS AND DISCUSSION

It is clear that the plasma motion modifies both the amplitude and the oscillatory phase of the reflected field (11); with a departure from the classical Snell's law ($\theta_i \neq \theta_r$). More precisely, (13) gives: $\tan\theta_r = \sin\theta_i/\gamma^2\left|\left(1+\beta^2\right)\cos\theta_i + 2\beta\right|$, which for large $\beta > 0$, predicts $\theta_r < \theta_i$, i.e. the reflection angle close to normal incidence. We note that earlier authors, Rattan et al 1973, erroneously performed inverse Fourier transform over the incident $\omega_i$; instead of integrating over the reflected frequency. The reflected waveforms for $E_{yR}$, as function of time and the plasma velocity $v\,(\mathbf{v} = \mathbf{e}_z v)$ for normal incidence ($\theta_i = 0$), are plotted in figure 2a. The time delays in terms of the inverse plasma frequency of the maximum positive and negative reflected amplitude, as a function of plasma velocity $\beta$, are shown in figure 2b. Large compression and

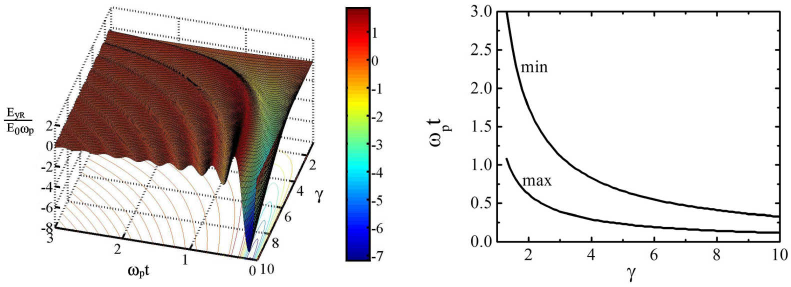


Figure 2: (a) Reflected EM field in time as a function of the plasma velocity $\gamma$. (b) Time period of the first and second peak in the reflected wave versus plasma velocity.

amplification of the reflected pulse (factor $\sim 2\gamma$) at highly relativistic plasma motion reveals a remarkable feature and some potential of this linear mechanism for ultra-short (attosecond) pulse generation by low intensity high-rep-rate femtosecond laser pulses scattering at counter-propagating relativistic electron beams, see e.g. Nikolić et al. 2008. For example, a short green laser light pulse ($\lambda \sim 0.5$ microns) reflected from 5MeV electrons ($\gamma \sim 10$) at critical density gives a main reflected pulse width of around 60 attoseconds; basically given by the relativistically upshifted plasma frequency which can be high in solid density plasmas.

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