Publ. Astron. Obs. Belgrade No. 84 (2008), 495 - 498 *Contributed Paper*

TRANSMITTANCE OF GRAVITO-ACOUSTIC WAVES IN NON-MAGNETIZED STRATIFIED PLASMA

V. M. ČADEŽ[1] and G. JOVANOVIĆ[2]

[1]*Astronomical Observatory Belgrade, Volgina 7, 11160 Belgrade, Serbia*
E–mail: vcadez@aob.bg.ac.yu

[2]*University of Montenegro, Natural-Science Faculty,*
POB 211, 81000 Podgorica, Montenegro
E–mail: gocaj@dell.pmf.cg.ac.yu

Abstract. We study transmittance properties of gravito-acoustic modes at a horizontal interface separating two isothermal regions of a gravitationally stratified non-magnetized plasma. Possible applications to the boundary between the solar interior and the corona are discussed.

1. INTRODUCTION

Observations and measurements of solar global oscillations have been carried out with high precision for decades (see Christensen-Dalsgaard 1989) within the framework of helioseismology studies, aiming to enlighten the internal structure of the Sun. The standard mathematical procedures in that field are based on solving eigen value problems for various models of solar interior and atmosphere (Goedbloed et al. 2004 and references therein). In this paper, we consider a driven problem in which wave modes propagate in a two region model of the system solar interior and solar atmosphere, the corona. The two different regions assumed quasi-isothermal and without magnetic fields are separated by a boundary z=0 where the considered gravito-acoustic waves suffer reflection and refraction. Taking physical parameters with values typical for a simplified two-region configuration of the solar photosphere and corona we analyze the conditions for gravito-acoustic waves to cross and totally reflect from the boundary z=0.

2. BASIC EQUATIONS

We start from the standard set of hydrodynamic equations describing the dynamics of adiabatic processes in a fully ionized hydrogen plasma in presence of gravity with constant acceleration:

$$
\begin{aligned}
&\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) \,=\, 0, \qquad \rho\frac{\partial\vec{v}}{\partial t} + \rho\vec{v}\cdot\nabla\vec{v} \,=\, -\nabla p + \rho\vec{g}, \\
&\frac{\partial p}{\partial t} + \vec{v}\cdot\nabla p \,=\, \gamma\frac{p}{\rho}\left(\frac{\partial\rho}{\partial t} + \vec{v}\cdot\nabla\rho\right).
\end{aligned} \tag{1}
$$

The above equations are linearized with respect to the initial basic state quantities Ψ_0 assuming small adiabatic perturbations $\Psi_1(x,y,z,t)$ that are harmonic in time t and in coordinates x and y, whose amplitudes $\hat{\Psi}_1$ satisfy the condition $|\hat{\Psi}_1| \ll |\Psi_0|$ and depend on the vertical coordinate z:

$$
\Psi_1(x,y,z,t) \,=\, \hat{\Psi}_1(k_x,k_y,\omega;z)\mathrm{e}^{-i\omega t + i(k_x x + k_y y)}. \tag{2}
$$

The unperturbed gas is initially in hydrostatic equilibrium and assumed to be stepwise isothermal T_0 =const, i.e. with constant speed of sound V_s in two regions separated by the boundary z=0. The basic state is thus described by:

$$
\frac{d}{dz}\ln\rho_0 \,=\, -\frac{\gamma g}{V_s^2} \tag{3}
$$

with $V_s^2 = \gamma \mathrm{RT}_0$, γ=$\mathrm{c}_p/\mathrm{c}_v$ being the ratio of specific heats; R=$\mathrm{R}_0/\overline{M}$ where R_0=8.3145 $\mathrm{JK}^{-1}\mathrm{mol}^{-1}$ is the universal gas constant and $\overline{M} = 0.5$kg mol^{-1} is the mean particle molar mass of the considered e-p plasma.

Eq. (3) now yields:

$$
\rho_0 \,=\, \rho_0(0)\mathrm{e}^{-z/H} \quad \text{where:} \quad H = \frac{V_s^2}{\gamma g}, \tag{4}
$$

while the linearized Eqs. (1) with perturbations given by Eq. (2) reduce to:

$$
\begin{aligned}
&V_s^2\omega^2\rho_0(z)\frac{d\xi_1}{dz} = g\omega^2\rho_0(z)\xi_1 - (\omega^2 - k_p V_s^2)p_1, \\
&V_s^2\frac{dp_1}{dz} = V_s^2\rho_0(z)(\omega^2 - \omega_{BV}^2)\xi_1 - gp_1,
\end{aligned} \tag{5}
$$

with $\rho_0(z)$ given by Eq. (4). Here, ξ_1 is defined by $v_{1z} = -i\omega\xi_1$, and:

$$
k_p^2 \equiv k_x^2 + k_y^2, \qquad \omega_{BV}^2 \equiv (\gamma - 1)\frac{g^2}{V_s^2},
$$

designate squares of the perpendicular wave-number and the Brunt-Väisäläa frequency respectively.

Eqs. (5) can further be transformed to a system of equations with constant coefficients if expressions:

$$
\exp(-z/2h)\xi_1 \quad \text{and} \quad \exp(z/2H)p_1
$$

are introduced as new unknowns. This finally yields the dispersion relation:

$$
k_p^2 = \frac{\omega^2 - \omega_{co}^2 - V_s^2 k_z^2}{\omega^2 - \omega_{BV}^2} \quad \frac{\omega^2}{V_s^2} \tag{6}
$$

where $\omega_{co}^2 \equiv \gamma^2 g^2/(4V_s^2)$ is the acoustic wave cut-off frequency squared.

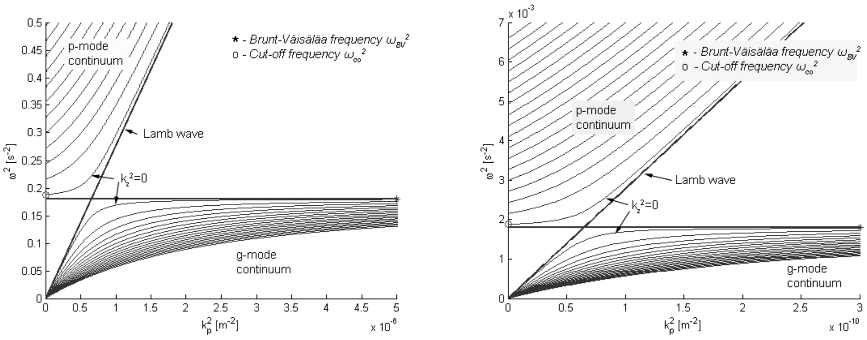

Figure 1: Dispersion curves for the region $z < 0$ (left) and region $z > 0$ (right) i.e. the photosphere and corona repsectively. Two sets of curves are related to acoustic p-modes and gravity g-modes with $k_z^2 = n \times 10^{-11} m^{-2}$ and $k_z^2 = n \times 10^{-11} m^{-2}$ (n=0,1,2...) in the left and right plot respectively.

3. RESULTS AND CONCLUSIONS

Figure 1 shows dispersion curves for the region 1 ($z < 0$) simulating the solar photosphere with temperature $T_1 = 10^4$K , and for the region 2 ($z > 0$) simulating the corona with temperature $T_2 = 10^6$K. Gravitational acceleration g is assumed constant within the domains of studied harmonic perturbations and we take $g = 274 ms^{-2}$ as it is on the Solar surface. Two discrete sets of (ω^2-k_p^2) curves are related to acoustic p-modes and gravity g-modes for $k_z^2 = n \times 10^{-11} m^{-2}$ and $k_z^2 = n \times 10^{-11} m^{-2}$ with n=0,1,2...

Figure 2 is a combination of two plots in Figure 1 showing domains of wave propagation where $k_z^2 > 0$ and wave evanescence where $k_z^2 < 0$ in both regions i.e. in the photosphere and corona. Three distinct ranges of the wave period τ and corresponding horizontal wavelength λ_p are noticeable in this figure:

1. Waves with periods $\tau > 200$s are transmitted through the boundary z=0 in both directions if their wavelength λ_p remains below certain value. Waves with larger λ_p are totally reflected in the corona, i.e. they do not enter the photosphere where they become evanescent. Further increase of λ_p results into evanescent waves in the corona too. Since waves become evanescent in both regions, this may indicate a possibility of surface modes at z=0.

2. For wave periods τ between 20s and 200s the waves can not be transmitted through the boundary z=0 as they are evanescent in one or in both regions depending on λ_p.

3. Waves with $\tau < 20$s are evanescent in both regions if λ_p is sufficiently small, then, at larger λ_p, they propagate only in the photosphere and are totally reflected from the boundary z=0. At even larger λ_p, these waves propagate in both regions, i.e. they are transmitted through z=0 between the photosphere and corona in both directions.

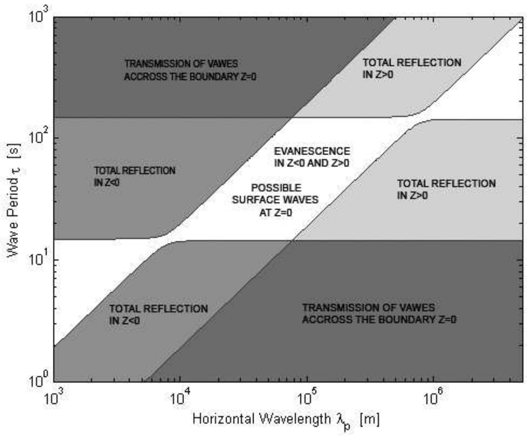

Figure 2: Domains of the bulk wave transmission, total reflection and total evanescence in the considered two region model.

As a conclusion, waves in the corona with sufficiently long, as well as those with short time periods can originate in the photosphere. An analogous statement holds for the wavelength intervals. For example, waves with the period τ=1000 s (≈15.7 min) can be transmitted through z=0 if $\lambda_p < 400$km.

In addition, we conclude that surface modes are possible at $z = 0$ for perturbations with $k_z^2 < 0$ in both regions around the boundary, i.e. in the photosphere and corona.

References

Christensen-Dalsgaard, J.: 1989, Stellar Oscillations (Aarhus, Lecture Notes, Astronomisk Institut, Aarhus Universitet.

Goedbloed, H., Poeds, S.: 2004, Principles of Magnetohydrodynamics,Cambridge University Press, Cambridge, UK.