Mathematicians vs Physicists: Disputes on Priority

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I. Newton: Amicus Plato amicus Aristoteles magis amica veritas. (English translation: Plato is my friend – Aristotle is my friend – but my greatest friend is truth.)

I. Newton (Letter to Robert Hooke (15 February 1676)) If I have seen further it is by standing on the shoulders of Giants.

Outline of the talk

- Hooke, Newton and Gravitation Law
- Poincare & Einstein: Special relativity and $E = mc^2$ law
- Hilbert & Einstein: Equations of gravitational field with matter
- Conclusions

Hooke, Newton and Gravitation Law

In 2007 it was 320th jubilee since publication "Mathematical Principles of Natural Philosophy" by I. Newton.

This book has established foundations of modern theoretical physics and (mathematical) calculus or foundations of modern mathematics.

I. Newton (1642 – 1727) is one of the greatest scientists in history.

Robert Hooke (1635 – 1703) is not so well-known. He started his scientific studies as an assistant of Boyle (who was an author of (Robert) Boyle (1627–1691) – (Edme) Mariotte (1620–1684) law). Basically, this law was discovered by Hooke and it was published in the Boyle book and Boyle quoted Hooke as a single discoverer and did not pretend to be even a co-author of the law.

Hooke was relatively poor man. He was a curator (secretary) of British Royal Society for forty years and his duties were to demonstrate experimental confirmations of new (3–4) discoveries at weekly Meetings of the Society. At the end of his life he counted about 500 discoveries.

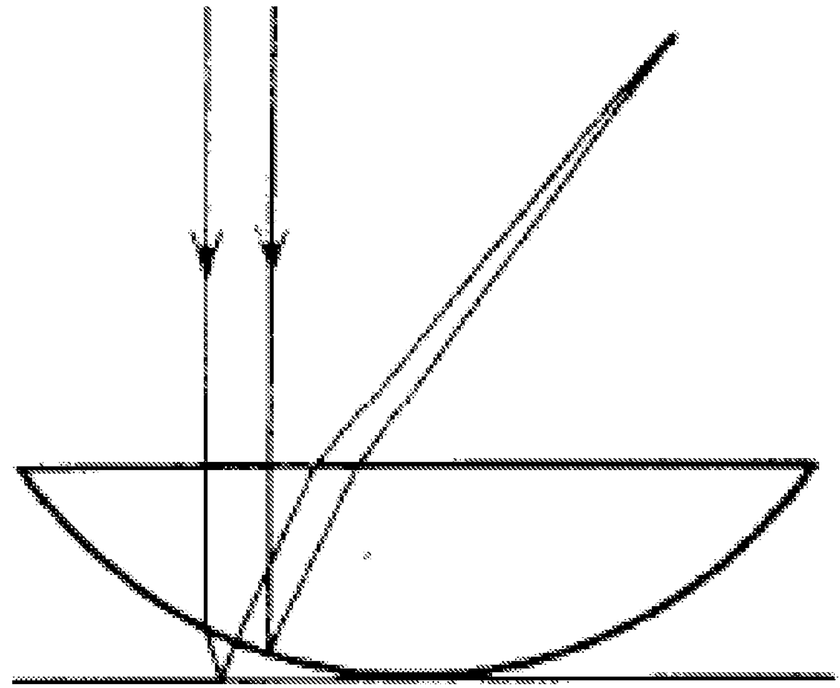
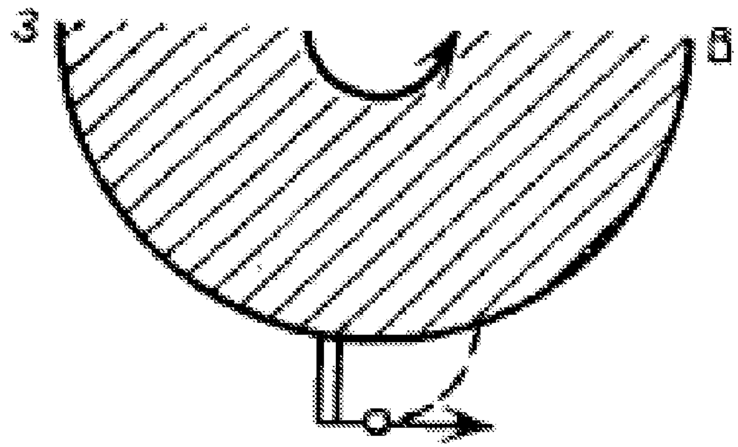


Figure 1: Newton's rings.



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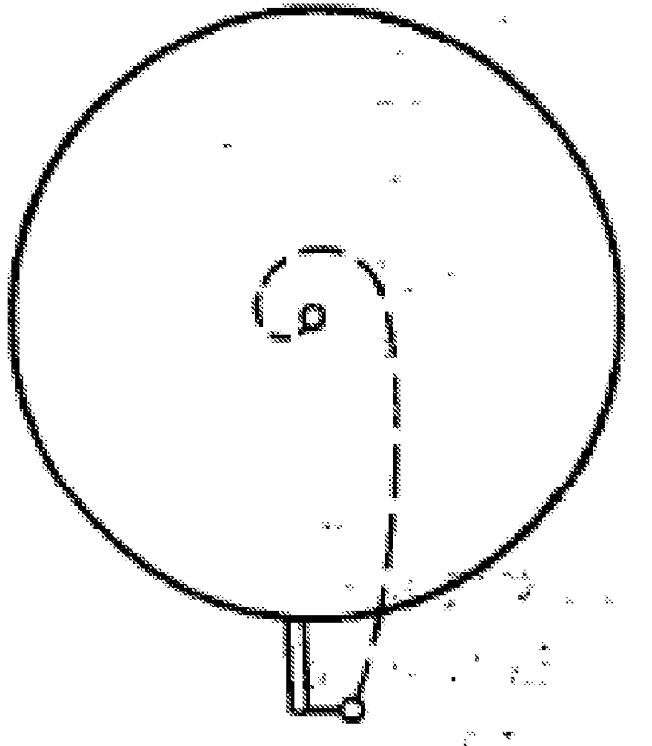

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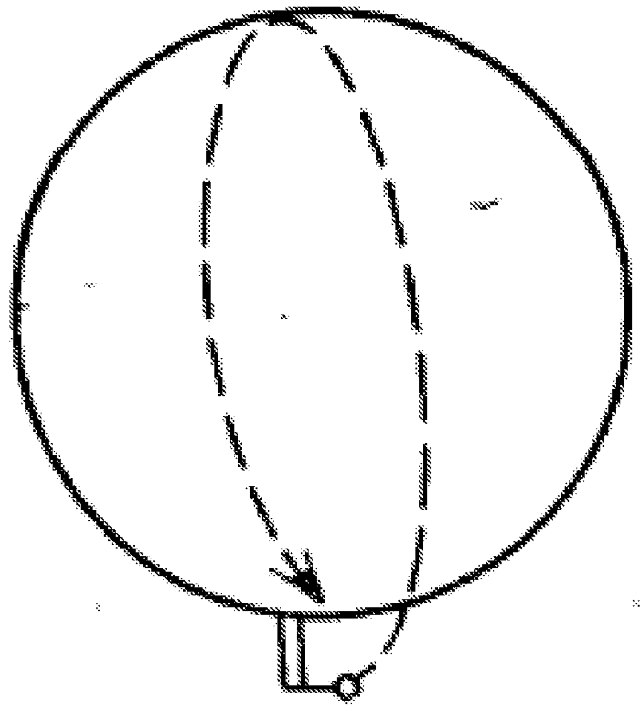

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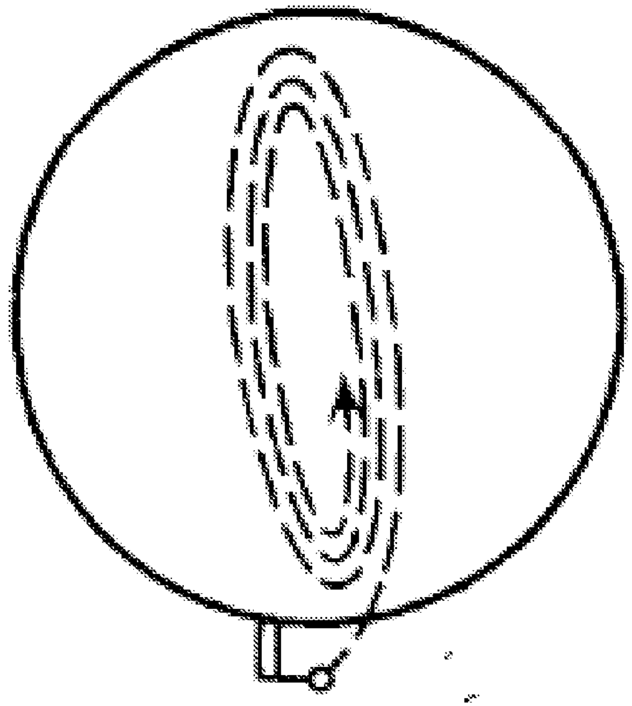

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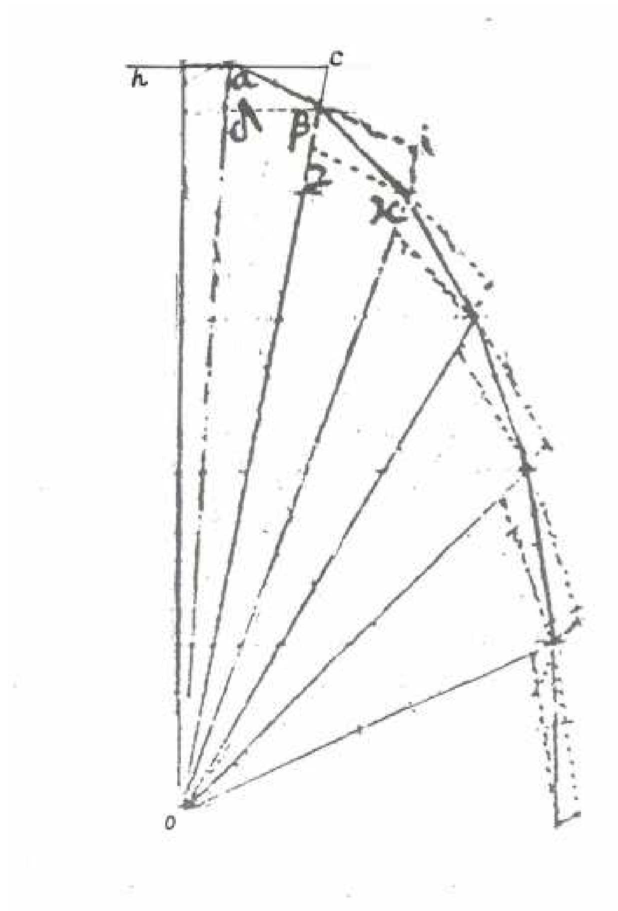


Figure 6: The upper right hand part of Hooke's Sept. 1685 diagram, with some auxiliary lines deleted, showing his geometrical construction for a discrete approximation to an elliptic orbit rotating clockwise under the action of a sequence of radial impulses which vary linearly with the distance from the center at O.

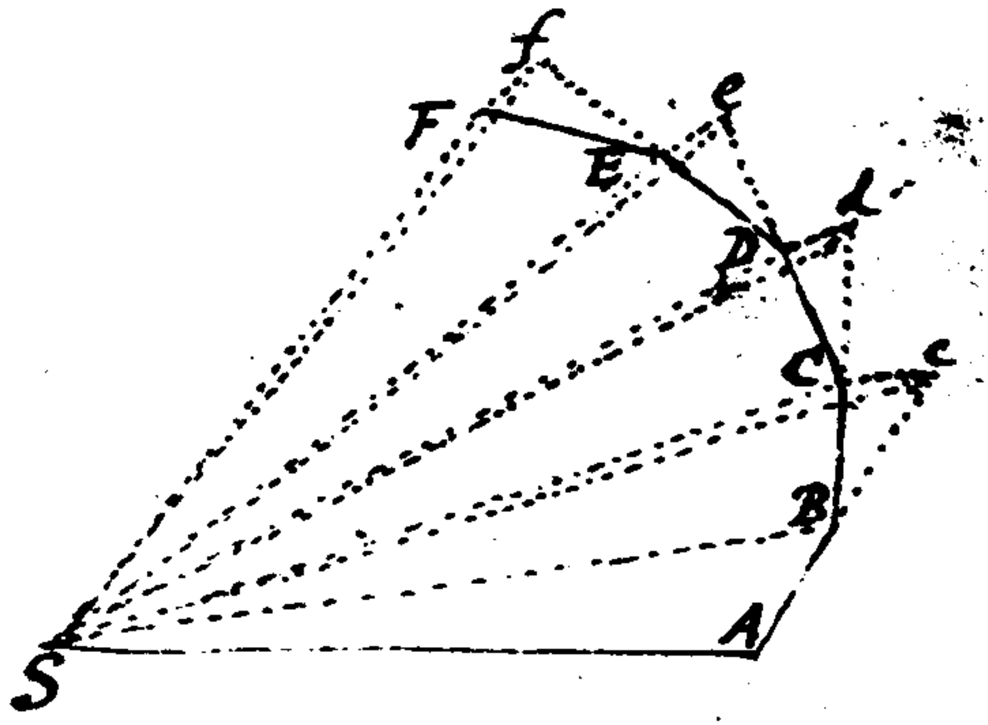


Figure 7: Diagram in De Motu associated with Newtons proof of Keplers area law, showing the construction of a discrete orbit rotating counterclockwise under the action of a sequence of radial impulses of unspecified magnitude with center at S.

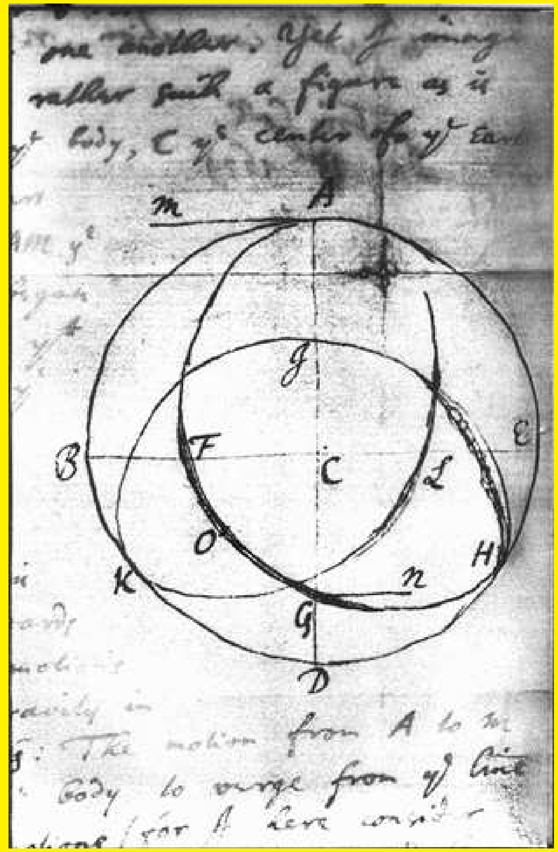


Figure 8: Diagram in Newtons Dec. 13, 1679 letter to Hooke, showing a curve AFOGHIKL for the approximate orbit of a body moving under the action of a constant central force.

In a brief handwritten but undated memorandum entitled "A True state of the Case and Controversy between Sir Isaac Newton & Dr Robert Hooke as the Priority of that Noble Hypothesis of Motion of ye Planets about ye Sun as their Centers", Hooke recounted his hypothesis for the physics of orbital motion and his theory of universal gravitation. Hookes memorandum, which remained unpublished during his lifetime, is historically quite accurate, contradicting numerous criticisms of his contemporaries and historians of science that Hooke always claimed for himself more credit than he actually deserved. In fact, to support his priority Hooke quoted verbatim from several extant documents: the transcript of his lecture on Planetary Movements as a Mechanical Problem given at the Royal Society on May 23, 1666, his short (28 pages) monograph, An Attempt to prove the motion of the Earth by Observations published in 1674, and his lengthy correspondence in the Fall of 1679 with Isaac Newton. However, Hooke did not mention his remarkable geometrical implementation of orbital motion for central force motion, see Fig. 8, based on the application of his physical principles, which was found only recently in a manuscript dated Sept. 1685.

Unfortunately, Hooke did not publish this manuscript and related work in spite of Edmond Halleys urging him "... that unless he produce another differing demonstration [from Newton's], and let the world judge of it, neither I nor any one else can believe it". It can be seen that Hookes geometrical construction is virtually the same as the one described by Newton, in connection with his proof of Kepler's area law in De Motu, a short draft that Newton sent to the Royal Society in 1684, which subsequently he expanded into his monumental work, the Principia.

In the first edition of the Principia Hooke's early proposal for universal gravitation was not mentioned, while in the second edition (1713), Newton left it to his editor, Roger Cotes, to admit in an editors preface, that the force of gravity is in all bodies universally others have suspected or imagined, but Newton was the first and only one who was able to demonstrate it from phenomena and to make it a solid foundation for his brilliant theories.

Even this small concession to others, was left out in Newton's third and final edition (1726) of the Principia. Apparently, after hearing of Hooke's priority complains, Newton eliminated many references to Hooke in earlier drafts of his Principia.

In a letter to Halley, Newton complained that

... he [Hooke] knew not how to go about it. Now is not this very fine? Mathematicians that find out, settle and do all the business must content themselves with being nothing but dry calculators & drudges and another that does nothing but pretend & grasp at all things must carry away all the invention as well as those who were to follow him as of those that went before him.

Poincare & Einstein: Special relativity

Poincare's contribution into Special relativity

In his book "Science and Hypothesis" (1902) H. Poincare noted:

"And now allow me to make a digression; I must explain why I do not believe, in spite of Lorentz, that more exact observations will ever make evident anything else but the relative displacements of material bodies. Experiments have been made that should have disclosed the terms of the first order; the results were nugatory. Could that have been by chance? No one has admitted this; general explanation was sought, and Lorentz found it. He showed that the terms of the first order should cancel each other, but not the terms of the second order. Then more exact experiments were made, which were also negative; neither could this be the result of chance. An explanation was necessary, and was forthcoming; they always are; hypotheses are what we lack the least. But this is not enough. Who is there who does not think that this leaves to chance far too important role? Would it not also be chance that this singular concurrence should cause certain circumstance to destroy the terms of the first order, and that totally different but very opportune circumstance should cause those of the second order to vanish? No; the same explanation must be found for the two cases, and everything tends to show that this explanation would serve equity well for the terms of the higher order, and that the mutual destruction of these terms will be rigorous and absolute."

In 1904, on the basis of experimental facts, H. Poincare generalized the Galilean relativity principle to all natural phenomena. He wrote:

"The principle of relativity, according to which the laws of physical phenomena should be the same, whether to an observer fixed, or for an observer carried along in a uniform motion of translation, so that we have not and could not have any means of discovering whether or not we are carried along in such a motion."

Just this principle has become the key one for the subsequent development of both electrodynamics and the theory of relativity. It can be formulated as follows. The principle of relativity is the preservation of form by all physical equations in any inertial reference system.

In the article "The theory of Lorentz and the principle of equal action and reaction", published in 1900, he wrote about the local time τ , defined as follows:

"I assume observers, situated at different points, to compare their clocks with the aid of light signals; they correct these signals for the transmission time, but, without knowing the relative motion they are undergoing and, consequently, considering the signals to propagate with the same velocity in both directions, they limit themselves to performing observations by sending signals from A to B and, then, from B to A. The local time τ is the time read from the clocks thus controlled. Then, if c is the velocity of light, and v is the velocity of the Earths motion, which I assume to be parallel to the positive X axis, we will have: $\tau = T - \frac{v}{c^2}X$".

In 1904, in the article "The present and future of mathematical physics", H. Poincare formulates the relativity principle for all natural phenomena, and in the same article he again returns to Lorentz's idea of local time. He writes:

"Let us imagine two observers, who wish to regulate their watches by means of optical signals; they exchange signals, but as they know that the transmission of light is not instantaneous, they are careful to cross them. When station B sees the signal from station A, its timepiece should not mark the same hour as that of station A at the moment the signal was sent, but this hour increased by constant representing the time of transmission. Let us suppose, for example, that station A sends it signal at the moment when its timepiece marks the hour zero, and that station B receives it when its time-piece marks the hour t. The watches will be set, if the time t is the time of transmission, and in order to verify it, station in turn sends signal at the instant when its time-piece is at zero; station must then see it when its time-piece is at t. Then the watches are regulated. And, indeed, they mark the same hour at the same physical instant, but under one condition, namely, that the two stations are stationary. Otherwise, the time of transmission will not be the in the two directions, since the station, for example, goes to meet the disturbance emanating from, whereas station flees before the disturbance emanating from A. Watches regulated in this way, therefore, will not mark the true time; they will mark what might be called the local time, so that one will gain on the other. It matters little, since we have means of perceiving it. All the phenomena which take place at, for example, will be behind time, but all just the amount, and the observer will not notice it since his watch is also behind time; thus, in accordance with the principle of relativity he will have means of ascertaining whether he is at rest or in absolute motion. Unfortunately this is not sufficient; additional hypotheses are necessary. We must admit that the moving bodies undergo a uniform contraction in the direction of motion".

H. Poincare discovered that Lorentz transformations, together with spatial rotations form a group.

H. Poincare (1905) discovered this group and named it the Lorentz group. He found the group generators and constructed the Lie algebra of the Lorentz group. Poincare was the first to establish that, for universal invariance of the laws of Nature with respect to the Lorentz transformations to hold valid, it is necessary for the physical fields and for other dynamical and kinematical characteristics to form a set of quantities transforming under the Lorentz transformations in accordance with the group, or, to be more precise, in accordance with one of the representations of the Lorentz group.

H. Poincare was the first to introduce the notion of four-dimensionality of a number of physical quantities.

H. Poincare discovered a number of invariants of the group and among these the fundamental invariant

$$J = c^2T^2 - X^2 - Y^2 - Z^2(*),$$

which arose in exploiting the Lorentz transformation. It testifies that space and time form a unique four-dimensional continuum of events with metric properties determined by the invariant (*). The four-dimensional space-time discovered by H. Poincare, and defined by invariant (*), was later called the Minkowski space.

Thus, depending on the choice of inertial reference system the projections X, Y, Z, T are relative quantities, while the quantity J for any given X, Y, Z, T has an absolute value. A positive interval J can be measured by a clock whereas a negative oneby a rod. According to (*), in differential form we have

$$(d\sigma)^2 = c^2(dT)^2 - (dX)^2 - (dY)^2 - (dZ)^2(**)$$

The quantity $d\sigma$ is called an interval. The geometry of space-time, i. e. the space of events (the Minkowski space) with the measure (3.23) has been termed pseudo-Euclidean geometry.

H. Poincare proved the invariance of Maxwell-Lorentz equations with 4-vector language.

H. Poincare formulated the invariance of Maxwell-Lorentz equations with 4-vector language and proved the famous $E = mc^2$ law (published in 1906).

- An outstanding British mathematician E. Whittaker was the first who came to the conclusion of the decisive contribution of H. Poincare to this problem when studying the history of creation of the special relativity theory, 50 years ago (Edmund Whittaker, A History of the Theories of Aether and Electricity (first edition 1910 ; revised edition vol. 1, The Classical Theories, 1951, vol. 2, The Modern Theories, 1900-1926, 1953), Thomas Nelson, 1962 and 1961). There is the chapter "The

Relativity Theory of Poincare and Lorentz". His monograph caused a remarkably angry reaction of some authors. But E. Whittaker was mainly right. H. Poincare really created the special theory of relativity grounding upon the Lorentz work of 1904 and gave to this theory a general character by extending it onto all physical phenomena.

- A very interesting debate took place on France-Culture the 22nd January 2005, between Jean-Paul Auffray and Jean Eisenstaedt, about the origins of the Theory of Relativity. Its current attribution to Albert Einstein has been called into question by specialists who are keen to take an objective standpoint, in spite of the glorification of this man by the media.

  "Today, weve got to really look at the evidence writes Allgre: "Einstein did not invent the (Special) Relativity Theory. The first one to discover it was the Frenchman Henri Poincare. Throughout the world, physics have known this ever since the Briton Edmund Whittaker said so, but few competent scientists wanted to check the truth of this fact. Nobody dared to question Einsteins absolute genius. Modern physics had put Einstein on a pedestal."

It is worth noting that Professor Allegre, Doctor in Physics, was also the Minister for State Education, Research and Technology between 1997 and 2000 and that he has been a member of the Academie des Sciences since 1995.

- Jean Eisenstaedt ("The curious history of Relativity"): "The English Physicist Sir Edmund Whittaker, a famous expert in relativity who in his scholarly book "Theories of Aether and Electricity" the chapter "The Relativity Theory of Poincare and Lorentz". Here Whittaker clearly showed his colors: Einstein is conspicuously absent."

- A. Pais had tried to prove on the pages of his book ("Subtle is the Lord: the science and the life of Albert Einstein", Oxford University Press, 1982) that H. Poincare had not made the decisive step to create the theory of relativity! He, a physicist, reinforced his view on the contribution of H. Poincare by the decision of the Paris Session of the French Philosophical Society in 1922. The philosophers have met and made a decision whereas they probably have not studied works by Poincare on the theory of relativity at all.

- There is a surprising statement by L. de Broglie (Nobel prize winner, Director of H. Poincare Institute) made in 1954:

  "A bit more and it would be H. Poincare, and not A. Einstein, who first built the theory of relativity in its full generality and that would deliver to French science the honor of this discovery.. . .But Poincare has not made the decisive step and left to Einstein the honor to uncover all the consequences following from the principle of relativity, and in particular, by means of a deep analysis of measurements of length and time, to discover the real physical nature of relation between space and time maintained by the principle of relativity".

$E = mc^2$ derivation

DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. EINSTEIN

September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:—

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

With these principles* as my basis I deduced *inter alia* the following result (§ 8):—

Let a system of plane waves of light, referred to the system of co-ordinates (x, y, z), possess the energy l; let the direction of the ray (the wave-normal) make an angle ϕ with the axis of x of the system. If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of co-ordinates in motion along the axis of x with the velocity v, then this quantity of light—measured in the system (ξ, η, ζ)—possesses the energy

$$l^* = l\frac{1 - \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}}$$

where c denotes the velocity of light. We shall make use of this result in what follows.

Let there be a stationary body in the system (x, y, z), and let its energy—referred to the system (x, y, z) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity v, be H_0.

Let this body send out, in a direction making an angle ϕ with the axis of x, plane waves of light, of energy $\frac{1}{2}\mathrm{L}$ measured relatively to (x, y, z), and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system (x, y, z). The principle of

*The principle of the constancy of the velocity of light is of course contained in Maxwell's equations.

energy must apply to this process, and in fact (by the principle of relativity) with respect to both systems of co-ordinates. If we call the energy of the body after the emission of light E_1 or H_1 respectively, measured relatively to the system (x, y, z) or (ξ, η, ζ) respectively, then by employing the relation given above we obtain

$$\begin{aligned} E_0 &= E_1 + \frac{1}{2}L + \frac{1}{2}L, \\ H_0 &= H_1 + \frac{1}{2}L\frac{1 - \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} + \frac{1}{2}L\frac{1 + \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} \\ &= H_1 + \frac{L}{\sqrt{1 - v^2/c^2}}. \end{aligned}$$

By subtraction we obtain from these equations

$$H_0 - E_0 - (H_1 - E_1) = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The two differences of the form $H - E$ occurring in this expression have simple physical significations. H and E are energy values of the same body referred to two systems of co-ordinates which are in motion relatively to each other, the body being at rest in one of the two systems (system (x, y, z)). Thus it is clear that the difference $H - E$ can differ from the kinetic energy K of the body, with respect to the other system (ξ, η, ζ), only by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies H and E. Thus we may place

$$\begin{aligned} H_0 - E_0 &= K_0 + C, \\ H_1 - E_1 &= K_1 + C, \end{aligned}$$

since C does not change during the emission of light. So we have

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of the electron (§ 10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2.$$

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2. The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that

The mass of a body is a measure of its energy-content; if the energy changes by L, the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

About this Document

This edition of Einstein's *Does the Inertia of a Body Depend upon its Energy-Content* is based on the English translation of his original 1905 German-language paper (published as *Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?*, in *Annalen der Physik*. **18**:639, 1905) which appeared in the book *The Principle of Relativity*, published in 1923 by Methuen and Company, Ltd. of London. Most of the papers in that collection are English translations by W. Perrett and G.B. Jeffery from the German *Das Relativatsprinzip*, 4th ed., published by in 1922 by Tuebner. All of these sources are now in the public domain; this document, derived from them, remains in the public domain and may be reproduced in any manner or medium without permission, restriction, attribution, or compensation.

The footnote is as it appeared in the 1923 edition. The 1923 English translation modified the notation used in Einstein's 1905 paper to conform to that in use by the 1920's; for example, c denotes the speed of light, as opposed the V used by Einstein in 1905. In this paper Einstein uses L to denote energy; the italicised sentence in the conclusion may be written as the equation "$m = L/c^2$" which, using the more modern E instead of L to denote energy, may be trivially rewritten as "$E = mc^2$".

This edition was prepared by John Walker. The current version of this document is available in a variety of formats from the editor's Web site:

http://www.fourmilab.ch/

Max Jammer

CONCEPTS
OF MASS

in classical and modern physics

HARVARD UNIVERSITY PRESS
CAMBRIDGE-MASSACHUSETTS, 1961

Figure 10: A fundamental book where it was point out that there is a logical loop in the first Einstein derivation of the famous formulae.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 42, NUMBER 8 AUGUST, 1952

Derivation of the Mass-Energy Relation

HERBERT E. IVES
Upper Montclair, New Jersey
(Received February 28, 1952)

The mass equivalent of radiation is implicit in Poincaré's formula for the momentum of radiation, published in 1900, and was used by Poincaré in illustrating the application of his analysis. The equality of the mass equivalent of radiation to the mass lost by a radiating body is derivable from Poincaré's momentum of radiation (1900) and his principle of relativity (1904). The reasoning in Einstein's 1905 derivation, questioned by Planck, is defective. He did not derive the mass-energy relation.

1. INTRODUCTION

THE equation relating mass to energy, $E=mc^2$, appears in two guises. In one guise it applies to radiation existing in space, and is applicable to the interaction of this radiation in pressure and impact phenomena where the radiation retains its identity as such. In these phenomena the "m" in the relation $E=mc^2$ is the *mass equivalent of free radiation*. In the second guise the relation $E=mc^2$ applies to radiation as emitted or absorbed by matter; in this case the "m" is the *mass of matter*, and the significance of the equation is that it describes the gain or loss of mass by matter when absorbing or emitting radiation. If we designate the two masses as m_R and m_M we then have *two* relations

$$E=m_Rc^2$$
$$E=m_Mc^2$$

to be established.

2. POINCARÉ AND THE MASS-ENERGY RELATION

In 1900, in a paper on "The Theory of Lorentz and the Principle of Reaction,"[1] H. Poincaré derived the expression $M=S/c^2$, where M is the *momentum* of radiation, S the *flux* of radiation, and c is the velocity of light. In explaining the significance of this momentum of radiation he said:

"Electromagnetic energy, from the point of view with which we are occupied, behaving like a fluid endowed with inertia, one must conclude that if any apparatus whatever, after having produced electromagnetic energy, sends it by radiation in a certain direction, the apparatus must recoil like a cannon which has launched a projectile ⋯. It is easy to calculate in figures the amount of this recoil. If the apparatus has a mass of one kilogram, and if it has sent in one direction, with the velocity of light, three millions of joules, the velocity due to the recoil is 1 cm per second."

Consider how Poincaré got this numerical result. He was using his formula, derived in this article, for the momentum of radiation, $M=S/c^2$, and he was putting down the expression for the conservation of momentum in the recoil process. Putting μ for the mass of the recoiling body, and v for its velocity, his working equation is then

$$\mu v=S/c^2.$$

For S, the energy flux, he put the energy E times c, the velocity of light. He then has

$$\mu v=S/c^2=Ec/c^2=E/c^2\cdot c.$$

Inserting his numerical values

$$\mu=10^3 \text{ grams}$$
$$E=3\times10^6 \text{ joules}=3\times10^{13} \text{ ergs}$$
$$c=3\times10^{10} \text{ cm per second,}$$

we get

$$10^3\times v=\frac{3\times10^{13}\times3\times10^{10}}{9\times10^{20}},$$

or

$$v=1.$$

The significant thing for our present study is that Poincaré in this calculation used E/c^2 for the coefficient of c in stating the momentum of radiation, that is, E/c^2 plays the role of mass. The relation $E=m_Rc^2$ was thus contained in his relation $M=S/c^2$.

Let us consider the nature of this "mass" of radiation. It follows from the pressure of radiation as deduced by Maxwell from his electromagnetic theory. Maxwell's formula

$$f=dE/cdt$$

describes the force exerted on an absorbing body by energy received at the rate dE/dt. Now force is also, by definition, the rate of change of momentum of the body, which, by the conservation of momentum, is also the rate of change of momentum of the radiation. We then have that the momentum lost by the radiation is equal to $1/c$ times the energy delivered to the body, or $M_R=E/c$. If now we designate the momentum of radiation by a "mass" m_R times the velocity of the radiation c, we have

$$m_Rc=E/c,$$

or

$$m_R=E/c^2.$$

We thus see that this "mass" of radiation is a concept derived through the definition of force as rate of change of momentum.

[1] H. Poincaré, Arch. néerland. sci. 2, 5, 232 (1900).

540

Figure 11: A fundamental article where it was pointed out that there is a logical loop in the first Einstein derivation of the famous formulae.

Poincaré stated that we may regard electromagnetic energy as a "fluide fictif" of density E/c^2. His momentum of energy is the density of this fictitious fluid times its velocity c. Poincaré discussed and rejected the idea that this fictitious mass was "indestructible," that is, that it was transferred entire in emission or absorption of energy. He decided it must appear as energy in other guises, and said "it is this which prevents us from likening completely this fictitious fluid to a real fluid." In our terminology Poincaré rejected the idea that m_R could become m_M.

In 1904 Poincaré formulated and named the "principle of relativity" according to which it is impossible by observations made on a body to detect its uniform motion of translation.[2] By the use of this principle it is possible to investigate the behavior of the "fluide fictif," and modify the conclusion of Poincaré just quoted.

Consider a body suspended loosely, as by a nonconducting cord, in the interior of an enclosure, the whole system being stationary with respect to the radiation transmitting medium. Let the body emit symmetrically in the "fore" and "aft" directions the amount of energy $\frac{1}{2}E$. The momenta of the two oppositely directed pulses cancel each other, the body does not move, and no information can be obtained as to its change of state.

Now let the whole system of enclosure and suspended particle be set in uniform motion with respect to the radiation transmitting medium with the velocity v. The body now possesses the momentum $mv/[1-(v^2/c^2)]^{\frac{1}{2}}$ and the problem is to determine the effect on this momentum of the two emitted wave trains. Now the energy contents of the two wave trains emitted for the same (measured) period of emission, taking into account the change of frequency of the source and the lengths of the trains,[3] are

$$\frac{E}{2}\frac{[1+(v/c)]}{[1-(v^2/c^2)]^{\frac{1}{2}}} \quad\text{and}\quad \frac{E}{2}\frac{[1-(v/c)]}{[1-(v^2/c^2)]^{\frac{1}{2}}}.$$

The accompanying momenta, from Poincaré's formula, are

$$\frac{E}{2c^2}\frac{[1+(v/c)]}{[1-(v^2/c^2)]^{\frac{1}{2}}}c \quad\text{and}\quad \frac{E}{2c^2}\frac{[1-(v/c)]}{[1-(v^2/c^2)]^{\frac{1}{2}}}c.$$

These being oppositely directed, the net imparted momentum is

$$Ev/c^2[1-(v^2/c^2)]^{\frac{1}{2}}.$$

Forming the equation for the conservation of momentum we have

$$\frac{mv}{[1-(v^2/c^2)]^{\frac{1}{2}}}=\frac{m'v'}{[1-(v'^2/c^2)]^{\frac{1}{2}}}+\frac{Ev}{c^2[1-(v^2/c^2)]^{\frac{1}{2}}},$$

where v' is the velocity of the body after the emission of the radiation.

Now according to Poincaré's principle of relativity, the body must behave in the moving system just as in the stationary system first considered, that is, it does not change its position or velocity with respect to the enclosure, hence $v'=v$, and we get

$$\frac{(m-m')v}{[1-(v^2/c^2)]^{\frac{1}{2}}}=\frac{Ev}{c^2[1-(v^2/c^2)]^{\frac{1}{2}}},$$

giving exactly

$$(m-m')=E/c^2,$$

a relation independent of v, and so holding for the stationary system. The radiating body loses mass E/c^2 when radiating mass E. This is the relation $E=m_Mc^2$.

It thus appears that the mass-energy relation in both its aspects, $E=m_Rc^2$ and $E=m_Mc^2$, is derivable rigorously from the work of Poincaré. His original position, that the mass of the "fluide fictif" must disappear to reappear in other forms of energy, need not have been taken. *Poincaré's inertia of radiation is conserved, and is transformable into or recoverable from mechanical mass.*

The above derivation of the relation $E=m_Mc^2$ was not given by Poincaré himself, and in fact this simple derivation is not met with until long after the relation in question had been established by other, more intricate derivations. The first explicit statement that the heat energy of a body increases its "mechanical" mass was made by F. Hasenöhrl in 1904. Hasenöhrl studied the problem of a hollow enclosure filled with radiation, to determine the effect of the pressure due to the radiation.[4] He showed that "to the mechanical mass of our system must be added an apparent mass $\mu=8E/3c^2$." This he later recalculated as $4E/3c^2$.[5] On the ground that the internal energy of a body must consist in part of radiation Hasenöhrl stated that in general the mass of a body will depend on its temperature.

In 1907 Planck made a more exhaustive study than Hasenöhrl's on the energy "confined" in a body, utilizing Poincaré's momentum of radiation.[6] He found that "through every absorption or emission of heat the inertial mass of a body alters, and the increment of mass

[2] H. Poincaré, "L'etat actuel et l'avenir de la physique mathematique," Congress of Arts and Sciences, St. Louis, Sept. 24, 1904; first published in full in La Revue des Idées, 80, Nov. 15, 1904. For appreciations of the pioneer contributions of Poincaré to the principle of relativity, the formulation of the Lorentz transformations, and the momentum of radiation, see articles by W. Wien and H. A. Lorentz in Acta Math. **38** (1921). See also, Ives, "Revisions of the Lorentz transformations," Proc. Am. Phil. Soc. **95**, 125 (1951).

[3] H. E. Ives, J. Opt. Soc. Am. **34**, 225 (1944).

[4] F. Hasenöhrl, Wien. Sitzungen **IIa**, 113 1039 (1904).

[5] F. Hasenöhrl, Ann. Physik, 4, 16, 589 (1905). Hasenöhrl noted that this increase of mass was identical with that found twenty years earlier by J. J. Thomson for the case of a charged spherical conductor in motion. For the transformation of the factor $\frac{4}{3}$ to unity upon considering the effect of the enclosure (shell), see Cunningham, *The Principle of Relativity* (Cambridge at the University Press, London, 1914), p. 189.

[6] M. Planck, Sitz. der preuss. Akad. Wiss., Physik. Math. Klasse. **13** (June, 1907).

TABLE I. Mass and energy relations.

		Before emission of radiation Subscript 0	After emission of radiation Subscript 1
Observed from x, y, z platform	Energy of body	E_0	$E_1 = E_0 - L$
	Radiated energy	0	L
	Mass of body	m	m'
	Momentum of body	0	0
	Kinetic energy of body	0	0
Observed from ξ, η, ζ platform	Energy of body	H_0	$H_1 = H_0 - L\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$
	Radiated energy	0	$L\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$
	Mass of body	$m\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$	$m'\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$
	Momentum of body	$mv\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$	$m'v\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$
	Kinetic energy of body	$K_0 = mc^2\left[1\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-1\right]$	$K_1 = m'c^2\left[1\Big/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-1\right]$

is always equal to the quantity of heat · · · divided by the square of the velocity of light in vacuo." This derivation of the relation $E = m_M c^2$ is historically the first valid and authentic derivation of the relation.

Recurring now to the simple derivation above, which depends only on Poincaré's momentum of radiation and his principle of relativity, this appears in an encyclopedic article by W. Pauli in 1920.[7] Since Pauli gives no reference for this treatment, in his otherwise very fully referenced article, it may be presumed to be original with him.[8] In 1933, Becker, in his revision of Abraham's text-book,[9] reproduces Pauli's treatment, with the following comment: "This example is especially of interest, because Einstein with its help derived for the first time the principle of the inertia of energy as a universal law." This comment is incorrect; Einstein, in the work referred to (1905), did not give this derivation; he did not use the momentum of radiation, which is an essential element of this "example," and his derivation was actually incompetent to give the result he announced. This is brought out in the Appendix to the present paper.

3. SUMMARY

Attention is called to the dual aspect of the relation $E = mc^2$, depending on whether the "m" refers to the mass equivalent of free radiation, or the mechanical mass gained or lost through the process of radiation. The first m, designated m_R, was disclosed by Poincaré in his presentation of the momentum of radiation. The second m, designated by m_M, can be obtained from Poincaré's momentum of radiation and his principle of relativity. Historically the first derivation of the relation $E = m_M c^2$ is to be ascribed to Hasenöhrl and Planck.

APPENDIX. THE 1905 DERIVATION BY EINSTEIN

In 1905 Einstein published a paper with the interrogatory title "Does the Inertia of a Body Depend upon its Energy Content",[10] a question already answered in the affirmative by Hasenöhrl. This paper, which has been widely cited as being the first proof of the "inertia of energy as such," describes an emission process by two sets of observations, in different units, the resulting equations being then subtracted from each other. It should be obvious *a priori* that the only proper result of such a procedure is to give $0 = 0$, that is, no information about the process can be so obtained. However the fallacy of Einstein's argument not having been heretofore explicitly pointed out, the following analysis is presented:

Einstein sought to derive the mass-energy relation by observing the loss of energy of a radiating body by two sets of observations, one made from a platform stationary with respect to the body, (x, y, z, system), the other from a platform moving with a uniform velocity v with respect to the body, (ξ, η, ζ, system). We shall use the symbols of Einstein's article, adding to them at the start, the accepted formulas for the kinetic energies. The latter are the crux of the problem. The symbols and their relations are most perspicuously set forth in Table I. The problem is to determine, from the data set forth in Table I, the relation between radiated energy (L) and the mass of

[7] W. Pauli, Jr., "Relativitätstheorie," Encyclopedia Math. Wiss. V-2, hft 4, 19, 679 (1920). Pauli assigns momentum to the body not by moving it but by observing it from a moving platform; however, the mathematical formulation is the same as in the treatment here given.

[8] J. Larmor, previously, in considering the case of a radiating body moving through space against the reaction of its own radiation, decided that it would continue at uniform velocity, by losing momentum, at the expense of mass E/c^2. ("On the dynamics of radiation," Proc. Intern. Congr. Math., Cambridge (1912), p. 213; and *Collected Works* (Cambridge University Press, London, England, 1920).

[9] R. Becker, *Théorie der Electronen* (B. G. Teubner, Leipzig, 1930-1933), p. 348.

[10] A. Einstein, Ann. Physik 18, 639 (1905).

the body before and after the emission of radiation (m and m').

We first review Einstein's derivation, accepting his values for the radiation as observed from the x, y, z, and ξ, η, ζ, platforms as L and $L/[1-(v^2/c^2)]^{\frac{1}{2}}$ (these values have been incorporated in the table), and forming by subtraction the equation

$$(H_0-E_0)-(H_1-E_1)=L\left[1\bigg/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-1\right].$$

After obtaining this equation Einstein introduces the kinetic energies with the statement "$\cdots$ it is clear that the differences $H-E$ can differ from the kinetic energy K of the body with respect to the other system only by an additional constant $C\cdots$ thus we may place

$$H_0-E_0=K_0+C$$
$$H_1-E_1=K_1+C,\text{"}$$

then

$$(H_0-E_0)-(H_1-E_1)=K_0-K_1,$$

and it follows that

$$(K_0-K_1)=L\left[1\bigg/\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}-1\right].$$

Neglecting magnitudes of the fourth and higher orders he then gets

$$K_0-K_1=\tfrac{1}{2}Lv^2/c^2.$$

This is the final equation of Einstein's paper. His conclusion as to its physical meaning, namely "if a body gives off the energy L in the form of radiation its mass diminishes by L/c^2" follows from an unstated step, namely

$$K_0-K_1=\tfrac{1}{2}(m-m')v^2,$$

so that

$$\tfrac{1}{2}(m-m')v^2=\tfrac{1}{2}Lv^2/c^2,$$

or

$$m-m'=L/c^2.$$

This is the relation $E=m_Mc^2$, which does not appear explicitly in the paper.

Now it is by no means "clear that, etc." Thus we find Planck in 1907, after deriving the relation in question, as already described, making the following comment:[11] "Einstein has already drawn essentially the same conclusion [Ann. Physik **18**, 639 (1905)] by the application of the relativity principle to a special radiation process, however under the assumption *permissible only as a first approximation*,[12] that the total energy of a body is composed additively of its kinetic energy and its energy referred to a system with which it is at rest."

[11] Reference 6, footnote on p. 566.
[12] My italics, H. E. I.

What Planck objected to was the relation

$$H-E=K+C,$$

or, as he states it

$$H=K+E+C,$$

where, as shown by reference to Table I, H and K are observed from one platform, E from another.

Let us look at this objection (which Planck did not follow up by explanatory analysis). We shall find that what Planck characterized as an assumption permissible only to a first approximation invalidates Einstein's derivation.

Take the relation above derived

$$(H_0-E_0)-(H_1-E_1)=L\left\{\frac{1}{[1-(v^2/c^2)]^{\frac{1}{2}}}-1\right\}.$$

From Table I we have

$$K_0=mc^2\left\{\frac{1}{[1-(v^2/c^2)]^{\frac{1}{2}}}-1\right\}$$

$$K_1=m'c^2\left\{\frac{1}{[1-(v^2/c^2)]^{\frac{1}{2}}}-1\right\},$$

so that

$$K_0-K_1=(m-m')c^2\left\{\frac{1}{[1-(v^2/c^2)]^{\frac{1}{2}}}-1\right\}.$$

By division,

$$(H_0-E_0)-(H_1-E_1)=\frac{L}{(m-m')c^2}(K_0-K_1),$$

which may be considered as the difference of the two relations

$$(H_0-E_0)=\frac{L}{(m-m')c^2}(K_0+C),$$

$$(H_1-{}_1E)=\frac{L}{(m-m')c^2}(K_1+C).$$

Now these are *not*

$$H_0-E_0=K_0+C$$
$$H_1-E_1=K_1+C.$$

They differ by the multiplying factor

$$L/(m-m')c^2.$$

What Einstein did by setting down these equations (as "clear") was to *introduce* the relation

$$L/(m-m')c^2=1.$$

Now this is the very relation the derivation was supposed to yield. It emerges from Einstein's manipulation of observations by two observers because it has been slipped in by the assumption which Planck questioned. The relation $E=m_Mc^2$ was not derived by Einstein.

A.A. Logunov

HENRI POINCARÉ
AND
RELATIVITY THEORY

Translated by G. Pontecorvo and V.O. Soloviev
edited by V.A. Petrov

Let us quote some extractions from the article by H. Poincaré published in 1900 "Lorentz theory and principle of equality of action and reaction" (put into modern notations by V. A. Petrov):

"First of all let us shortly remind the derivation proving that the principle of equality of action and reaction is no more valid in the Lorentz theory, at least when it is applied to the matter.

We shall search for the resultant of all ponderomotive forces applied to all electrons located inside a definite volume. This resultant is given by the following integral

$$\vec{F} = \int \rho dV \left(\frac{1}{c}[\vec{v}, \vec{H}] + \vec{E} \right),$$

where integration is over elements dV of the considered volume, and $\vec{v}$ is the electron velocity.

Due to the following equations

$$\frac{4\pi}{c}\rho\vec{v} = -\frac{1}{c} \cdot \frac{\partial \vec{E}}{\partial t} + \mathrm{rot}\, \vec{H},$$
$$4\pi\rho = \mathrm{div}\vec{E},$$

and by adding and subtracting the expression $\frac{1}{8\pi}\nabla H^2$, *I can write the following formula*

$$\vec{F} = \sum_1^4 \vec{F}_i,$$

where

$$\vec{F}_1 = \frac{1}{4\pi c}\int dV\left[\vec{H}\frac{\partial\vec{E}}{\partial t}\right],$$
$$\vec{F}_2 = \frac{1}{4\pi}\int dV(\vec{H}\nabla)\vec{H},$$
$$\vec{F}_3 = -\frac{1}{8\pi}\int dV\nabla H^2,$$
$$\vec{F}_4 = \frac{1}{4\pi}\int dV\vec{E}(\mathrm{div}\vec{E}).$$

Integration by parts gives the following

$$\vec{F}_2 = \frac{1}{4\pi}\int d\sigma\vec{H}(\vec{n}\vec{H}) - \frac{1}{4\pi}\int dV\vec{H}(\mathrm{div}\vec{H}),$$
$$\vec{F}_3 = -\frac{1}{8\pi}\int d\sigma\vec{n}H^2,$$

where integrals are taken over all elements $d\sigma$ of the surface bounding the volume considered, and where $\vec{n}$ denotes the normal vector to this element. Taking into account

$$\mathrm{div}\vec{H} = 0,$$

it is possible to write the following

$$\vec{F}_2 + \vec{F}_3 = \frac{1}{8\pi}\int d\sigma\Big(2\vec{H}(\vec{n}\vec{H}) - \vec{n}H^2\Big). \qquad (A)$$

Now let us transform expression $\vec{F}_4$. Integration by parts gives the following

$$\vec{F}_4 = \frac{1}{4\pi}\int d\sigma\vec{E}(\vec{n}\vec{E}) - \frac{1}{4\pi}\int dV(\vec{E}\nabla)\vec{E}.$$

Let us denote two integrals from r.h.s. as $\vec{F}_4'$ and $\vec{F}_4''$, then

$$\vec{F}_4 = \vec{F}_4' - \vec{F}_4''.$$

Accounting for the following equations

$$[\nabla\vec{E}] = -\frac{1}{c}\cdot\frac{\partial\vec{H}}{\partial t},$$

we can obtain the following formula

$$\vec{F}_4'' = \vec{Y} + \vec{Z},$$

where

$$\vec{Y} = \frac{1}{8\pi}\int dV\nabla E^2,$$

$$\vec{Z} = \frac{1}{4\pi c}\int dV\left[\vec{E}\,\frac{\partial\vec{H}}{\partial t}\right].$$

As a result we find that

$$\vec{Y} = \frac{1}{8\pi}\int d\sigma\vec{n}E^2,$$

$$\vec{F}_1 - \vec{Z} = \frac{d}{dt}\int\frac{dV}{4\pi c}[\vec{H}\,\vec{E}].$$

At last we get the following

$$\vec{F} = \frac{d}{dt}\int\frac{dV}{4\pi c}[\vec{H}\,\vec{E}] + (\vec{F}_2 + \vec{F}_3) + (\vec{F}_4' - \vec{Y}),$$

where $(\vec{F}_2 + \vec{F}_3)$ *is given by Eq. (A), whereas*

$$\vec{F}_4' - \vec{Y} = \frac{1}{8\pi}\int d\sigma\Big(2\vec{E}(\vec{n}\vec{E}) - \vec{n}E^2\Big).$$

Term $(\vec{F}_2+\vec{F}_3)$ *represents the pressure experienced by different elements* $d\sigma$ *of the surface bounding the volume considered. It is straightforward to see that this* *pressure is nothing else, but the Maxwell* ***magnetic pressure*** *introduced by this scientist in well-known theory. Similarly, term* $(\vec{F}_4' - \vec{Y})$ *represents action of the Maxwell electrostatic pressure. In the absence of the first term,*

$$\frac{d}{dt}\int dV \frac{1}{4\pi c}[\vec{H}\,\vec{E}]\,,$$

the ponderomotive force would be nothing else, but a result of the Maxwell pressures. If our integrals are extended on the whole space, then forces $\vec{F}_2, \vec{F}_3, \vec{F}_4'$ *and* $\vec{Y}$ *disappear, and the rest is simply*

$$\vec{F} = \frac{d}{dt}\int \frac{dV}{4\pi c}[\vec{H}\,\vec{E}].$$

If we denote as M *the mass of one of particles considered, and as* $\vec{v}$ *its velocity, then we will have in case when the principle of equality of action and reaction is valid the following:*

$$\sum M\vec{v} = \text{const.}$$ [3]

Just the opposite, we will have:

$$\sum M\vec{v} - \int \frac{dV}{4\pi c}[\vec{H}\,\vec{E}] = \text{const.}$$

Let us notice that

$$-\frac{c}{4\pi}[\vec{H}\,\vec{E}]$$

is the Poynting vector of radiation.

[3]The matter only is considered here. — *A. L.*

If we put

$$J = \frac{1}{8\pi}(H^2 + E^2),$$

then the Poynting equation gives the following

$$\int \frac{dJ}{dt} dV = -\int d\sigma \frac{c}{4\pi} \vec{n}[\vec{H}\,\vec{E}] - \int dV \rho(\vec{v}\vec{E}). \quad (B)$$

The first integral in the r.h.s., as well known, is the amount of electromagnetic energy flowing into the considered volume through the surface and the second term is the amount of electromagnetic energy created in the volume by means of transformation from other species of energy.

We may treat the electromagnetic energy as a fictitious fluid with density J which is distributed in space according to the Poynting laws. It is only necessary to admit that this fluid is not indestructible, and it is decreasing over value $\rho dV \vec{E}\vec{v}$ in volume element dV in a unit of time (or that an equal and opposite in sign amount of it is created, if this expression is negative). This does not allow us to get a full analogy with the real fluid for our fictitious one. The amount of this fluid which flows through a unit square surface oriented perpendicular to the axis i, at a unit of time is equal to the following

$$JU_i,$$

where U_i are corresponding components of the fluid velocity.

Comparing this to the Poynting formulae, we obtain

$$J\vec{U} = \frac{c}{4\pi}[\vec{E}\,\vec{H}];$$

so our formulae take the following form

$$\sum M\vec{v} + \int dV \frac{J\vec{U}}{c^2} = \text{const.}^{\,4} \qquad (C)$$

They demonstrate that the momentum of substance plus the momentum of our fictitious fluid is given by a constant vector.

In standard mechanics one concludes from the constancy of the momentum that the motion of the mass center is rectilinear and uniform. But here we have no right to conclude that the center of mass of the system composed of the substance and our fictitious fluid is moving rectilinearly and uniformly. This is due to the

[4]In Eq. (C) the second term in the l.h.s. determines the total momentum of the electromagnetic radiation. Just here the concept of *radiation momentum density* arises

$$\vec{g} = \frac{J}{c^2}\vec{U},$$

and also the concept of *mass density of the electromagnetic field*

$$m = \frac{J}{c^2},$$

where J is the electromagnetic energy density. It is also easy to see from here that radiation energy density

$$\vec{S} = \frac{c}{4\pi}[\vec{E}\,\vec{H}]$$

is related to the momentum density

$$\vec{g} = \frac{\vec{S}}{c^2}.$$

So the notions of local *energy* and *momentum* appeared. All this was firstly obtained by H. Poincaré. Later these items were discussed in the Planck work (Phys. Zeitschr. 1908. **9.** S. 828) — *A. L.*

fact that this fluid is not indestructible.

The position of the mass center depends on value of the following integral

$$\int \vec{x} J dV,$$

which is taken over the whole space. The derivative of this integral is as follows

$$\int \vec{x}\frac{dJ}{dt}dV = -\int \vec{x}\mathrm{div}(J\vec{U})dV - \int \rho\vec{x}(\vec{E}\vec{v})dV.$$

But the first integral of the r.h.s. after integration transforms to the following expression

$$\int J\vec{U}dV$$

or

$$\left(\vec{C} - \sum M\vec{v}\right)c^2,$$

when we denote by $\vec{C}$ the constant sum of vectors from Eq. (C).

Let us denote by M_0 the total mass of substance, by $\vec{R}_0$ the coordinates of its center of mass, by M_1 the total mass of fictitious fluid, by $\vec{R}_1$ its center of mass, by M_2 the total mass of the system (substance + fictitious fluid), by $\vec{R}_2$ its center of mass, then we have

$$M_2 = M_0 + M_1, \quad M_2\vec{R}_2 = M_0\vec{R}_0 + M_1\vec{R}_1,$$

$$\int \vec{x}\frac{J}{c^2}dV = M_1\vec{R}_1 .^5$$

[5] H. Poincaré also exploits in this formula the concept of the *mass density of the electromagnetic field* introduced by him earlier. — *A. L.*

Then we come to the following equation

$$\frac{d}{dt}(M_2\vec{R}_2) = \vec{C} - \int \vec{x}\frac{\rho(\vec{v}\vec{E})}{c^2}dV. \qquad (D)$$

Eq. (D) may be expressed in standard terms as follows. If the electromagnetic energy is created or annihilated nowhere, then the last term disappears, whereas the center of mass of the system formed of the matter and electromagnetic energy (treated as a fictitious fluid) has a rectilinear and uniform motion”.

Then H. Poincaré writes:

“So, the electromagnetic energy behaves as a fluid having inertia from our point of view. And we have to conclude that if some device producing electromagnetic energy will send it by means of radiation in a definite direction, then this device must experience a recoil, as a cannon which fire a shot. Of course, this recoil will be absent if the device radiates energy isotropically in all directions; just opposite, it will be present when this symmetry is absent and when the energy is emitted in a single direction. This is just the same as this proceeds, for example, for the H. Hertz emitter situated in a parabolic mirror. It is easy to estimate numerically the value of this recoil. If the device has mass 1 kg, and if it sends three billion Joules in a single direction with the light velocity, then the velocity due to recoil is equal to 1 sm/sec”.

When determining the velocity of recoil H. Poincaré again exploits the formula

$$M = \frac{E}{c^2}.$$

In §7 of article [3] H. Poincaré derives equations of relativistic mechanics. If we change the system of units in this paragraph from $M = 1, c = 1$ to Gaussian system of units, then it is easy to see that **inert mass of a body** is also determined by formula:

$$M = \frac{E}{c^2}.$$

Therefore, it follows from works by H. Poincaré that the **inert mass** both of **substance**, and of **radiation** is determined by their energy. All this has been a consequence of the electrodynamics and the relativistic mechanics.

In 1905 Einstein has published the article "Does the inertia of a body depend on the energy contained in it?". Max Jammer wrote on this article in his book "The concept of mass in classical and modern physics" (Harvard University Press, 1961.):

> ≪*It is generally said that "the theorem of inertia of energy in its full generality was stated by Einstein (1905)" (Max Born.* "Atomic physics". Blackie, London, Glasgow ed. 6, p. 55*). The article referred to is Einstein's paper,* "Does the inertia of a body depend upon its energy content?". *On the basis of the Maxwell-Hertz equations of the electromagnetic field Einstein contended that "if a body gives off the energy E in the form of radiation, its mass diminishes by E/c^2". Generalizing this result for all energy transformations Einstein concludes: "The mass of a body is a measure of its energy content".*
>
> *It is a curious incident in the history of scientific thought that Einstein's own derivation of formula $E = mc^2$, as published in his article in the* "Annalen der Physik", *was basically fallacious. In fact, what for the layman is known as "the most famous mathematical formula ever projected" in science (*William Cahn.

"Einstein, a pictorial biography". New York: Citadel. 1955. P. 26*) was but the result of* *petitio principii*, *the conclusion of begging the question≫.*

"The logical illegitimacy of Einstein's derivation has been shown by Ives (Journal of the Optical Society of America. 1952. **42**, pp. 540-543)".

Let us consider shortly Einstein's article of 1905 "Does the inertia of a body depend on the energy contained in it?" Einstein writes:

"Let there be a body at rest in the system (x, y, z), whose energy, referred to the system (x, y, z), is E_0. The energy of the body with respect to the system (ζ, η, ς), which is moving with velocity v as above, shall be H_0.

Let this body simultaneously emit plane waves of light of energy $L/2$ (measured relative to (x, y, z)) in a direction forming an angle φ with the x-axis and an equal amount of light in the opposite direction. All the while, the body shall stay at rest with respect to the system (x, y, z). This process must satisfy the energy principle, and this must be true (according to the principle of relativity) with respect to both coordinate systems. If E_1 and H_1 denote the energy of the body after the emission of light, as measured relative to systems (x, y, z) and (ζ, η, ς), respectively, we obtain, using the relation indicated above,

$$E_0 = E_1 + \left[\frac{L}{2} + \frac{L}{2}\right],$$

$$H_0 = H_1 + \left[\frac{L}{2} \cdot \frac{1 - \frac{v}{V}\cos\varphi}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} + \frac{L}{2} \cdot \frac{1 + \frac{v}{V}\cos\varphi}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} \right] =$$

$$= H_1 + \frac{L}{\sqrt{1 - \left[\frac{v}{V}\right]^2}}.$$

Subtracting, we get from these equations

$$(H_0 - E_0) - (H_1 - E_1) = L \left\{ \frac{1}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} - 1 \right\}". \quad (N)$$

A. Einstein tries to get all the following just from this relation. Let us make an elementary analysis of the equation derived by him. According to the theory of relativity

$$H_0 = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad H_1 = \frac{E_1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Einstein seemingly did not take into account such formulae. It follows then that

$$H_0 - E_0 = E_0 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right), \quad H_1 - E_1 = E_1 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right),$$

and consequently the l.h.s. of the Einstein equation is equal to the following

$$(H_0 - E_0) - (H_1 - E_1) = (E_0 - E_1) \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right);$$

then Eq. (N) takes an apparent form

$$E_0 - E_1 = L.$$

Therefore, it is impossible to get something more substantial from the initial Einstein equation (N). In this work A. Einstein has not succeeded in discovering neither physical arguments, nor a method of calculation to prove that formula

$$M = \frac{E}{c^2}$$

is valid at least for radiation. So, the critics given by Ives on the A. Einstein work is correct. In 1906 Einstein once more returns to this subject, but his work reproduces the Poincaré results of 1900, as he notes himself.

Later, Planck in 1907 and Langevin in 1913 revealed, on this basis, the role of internal interaction energy (binding energy), which **led to the mass defect, providing conditions for possible energy release, for example, in fission and fusion of atomic nuclei**. The relativistic mechanics has become an engineering discipline. Accelerators of elementary particles are constructed with the help of it.

"Disproofs" of the special theory of relativity appearing sometimes are related to unclear and inexact presentation of its basics in many textbooks. Often its meaning is deeply hidden by plenty of minor or even needless details presented. The special theory of relativity is strikingly simple in its basics, almost as Euclidean geometry.

On the transformations of force

According to (9.8) and (9.11) the four-force is 50

$$F^\nu = \left(\gamma \frac{\vec{v}\vec{f}}{c}, \gamma\vec{f}\right), \tag{9.19}$$

Hilbert & Einstein: Equations of gravitational field with matter

One of the early references is the book Einstein, Hilbert and the Theory of Gravitation by a renowned historian of science J. Mehra (Einstein, Hilbert and the Theory of Gravitation. D. Reidel. Publishing Company, Dordrecht, Holland, Boston (1974)), where the great role of Hilbert was showed very clear. Such view was strengthened in 1978 when the correspondence between Einstein and Hilbert was published, from which followed that Hilbert informed Einstein on the gravitational field equations in a letter before his formal publication (J. Earman, and C. Glymour, Einstein and Hilbert: Two months in the history of general relativity, Arch. Hist. Exact. Sci. 19 (1978) 291)).

However, in 1997 a new sensation shaked just established opinion:

the authors of a short article in "Science" (L. Corry, J. Renn, J. Stachel, Belated Decision in the Hilbert-Einstein Priority Dispute, Science 278 (1997) 1270) argued on the basis of the first proofs of the Hilbert paper on the gravitational equations, digged up from the Hilbert archive, that Hilbert had no correct, generally covariant equation before Einstein. Moreover, the authors of transparently alluded that Hilbert "borrowed" some decisive formulae from Einstein! And even that Hilbert tried to hide such an appropriation with help of deliberately wrong dating of his article. Such an accusation would seriously undermine the image of David Hilbert from the ethical side, and was in a sharp contrast to all what was known about his personality.

In his fundamental article D. Hilbert derived gravitational field equations from chosen Lagrangian.

Hilbert–Einstein–General Relativity

In late June - early July 1915 Einstein spends a week in Göttingen where (as he witnesses in a letter to Zangger of 7 July) he gave six two-hour lectures thereh. By all accounts he seems happy with the outcome:

"To my great joy, I succeeded in convincing Hilbert and Klein completely" (E. to de Haas), "I am enthusiastic about Hilbert" (E. to Sommerfeld). The feelings appear to be mutual. Hilbert recommends Einstein for the third Bolyai Prize in 1915 for the high mathematical spirit of his achievements(the first and the second recipients of the Bolyai prize have been Poincare and Hilbert).

Nevertheless, the Göttingen discussions seem to have reinforced Einsteinfs uneasiness about the lack of general covariance of his (and Grossmann) equations. He is reluctant (he writes to Sommerfeld in July 1915) to include his papers on general relativity in a new edition of "The Principle of Relativity", "because none of the presentations to date is complete".

After the November race Einstein will state more precisely (in letters to friends) the grounds for his discontent with the old theory:

(1) its restricted covariance did not include uniform rotations;

(2) the precession of the perihelion of Mercury came out 18" instead of the observed 45" per century;

(3) his proof of October 1914 of the uniqueness of the gravitational Hamiltonian is not correct. In the meantime Einstein receives a letter by Sommerfeld (perhaps in late October 1915 . the letter is lost) from which he learns that he is not the only one dissatisfied with his 1914 theory.

Hilbert also has objections to it and is working on his own on "Die

Grundlagen der Physik" originally conceived as "Die Grundgleichungen /basic equations/ der Physik".

Will Einstein let someone else, be it Hilbert himself, share with him the fruit of years of hard work and great inspiration? Not he! At 36, he can still fight. The Einstein papers reveal an unprecedented activity in November 1915. Einstein submits four communications to the "Preussische Akademie der Wissenschaften": on 4, 11, 18 and 25 November, no Thursday is skipped! These are not different parts of a larger work.

The first, "Zur allgemeine RelativitNatstheorie" rejects his formulation of 1914 and proposes a new fundamental equation.

The second, with the same title, rejects the first and starts anew.

The fourth, "Die Feldgleichungen der Gravitation" rejects the first two and finally contains the right equations. It is like in a movie when the film is turned on a high speed. Nothing similar has happened either before or after in Einsteinfs life. But this is not all. Einstein only answers (the lost) Sommerfeld's letter on 28 November (three days after his last talk at the Academy). "Don't be angry with me"... he writes "for only today answering your friendly and interesting letter". But last month I had one of the most exciting, most strenuous times of my life, also one of the most rewarding. I could not concentrate on writing". Indeed, from late October to late November Einstein stops writing to any of his habitual addressees: Besso, Ehrenfest, Lorentz, . . . But he does write letters (or, rather, postcards).

He only replaces all his regular correspondents by a single new one – Hilbert. Four postcards are preserved from Einstein to Hilbert dated 7, 12, 15, 18 November and two of the four Hilbert answers.

On 7 November Einstein sends to Hilbert the proofs of his November-four paper and in the accompanying card writes "I recognized four weeks ago that my earlier methods of proof were deceptive". He alludes to the above mentioned letter of Sommerfeld which reports on Hilbertfs objections to the October 1914 paper; and closes by saying: "I am curious whether you will be well disposed towards this solution".

Hilbert would have hardly been well disposed towards the new equation, since it assumes that the determinant of the metric tensor is a constant (–1) and is hence still not generally covariant. Probably, after having Hilbertfs criticism (which has been lost) Einstein opted on 11 November for the generally covariant equation

$R_{\mu\nu} = \kappa T_{\mu\nu}$, (Einstein–Grossmann)

which Grossmann and he have rejected two years earlier.

It only coincides, however, with the correct equation (1) if $T_{\mu\nu}$ (and hence also $T_{\mu\nu}$) is traceless. This is the case of Maxwell electrodynamics and Einstein speculates that it may be more general.

The next day, 12 November, Einstein sends a second postcard to Hilbert announcing that he had finally achieved generally covariant field equations. He also thanks Hilbert for his "kind letter" (which is lost). Hilbert replies on 14 November a long message on two postcards. He is excited about his own "axiomatic solution of your grand problem". In a postscript Hilbert adds that his theory is "wholly distincth from Einsteinfs and invites Einstein to come to Göttingen and hear his lecture on the subject. The tone is cordial: Hilbert urges Einstein to come to Göttingen the day before the lecture and pass the night at Hilbert's home. The next day, Monday, 15 November, Einstein already answers Hilbert's cards. (One cannot fail to notice how accurately the mail service is working in Germany in the midst of the European war.) "The indications on your postcards lead to the greatest expectations".

He apologizes for his inability to attend the lecture, since he is overtired and bothered by stomach pains. Asks for a copy of the proofs of Hilbert's paper. Apparently, he does receive the requested copy within three days, because on 18 November, the day of his third talk at the Academy, Einstein writes his fourth postcard: "The system [of equations] given by you agrees – as far as I can see – exactly with what I found in recent weeks and submitted to the Academy". Then Einstein remarks that he has known about Eq.(Einstein–Grossmann) "for three years" but that he and Grossmann have rejected it on the grounds that in the Newtonian limit they are not compatible with "Newton's law" (meaning Poisson's field equation). Finally, Einstein informs Hilbert that he is finally explaining the advance of the perihelion of Mercury from general relativity alone without the aid of any subsidiary hypotheses. Two remarks are in order.

First, it is not true that Hilbert's

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}, \text{ (Hilbert)}$$

is equivalent to Einstein's Eq. of the paper submitted to the Academy on 11 November. (It will be equivalent to the equation Einstein is going to write a week later. It seems, however, that Einstein does have in mind his Eq. in this postcard since he is adding the priority claim that he knew it for three years.) The two equations are only consistent with one another for $T (= T^{\nu}_{\nu}) = 0$, the case Einstein has been mostly interested in at the time.

Second, Einstein does derive the correct value for the advance of the perihelion of Mercury in his third communication "Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie" from his not exactly correct equation. This is possible since he is actually solving the homogeneous equation (with $T_{\mu\nu} = 0$) in the post Newtonian approximation (allowing for point singularities). - In seeing the physical implications of the theory Einstein has no competitor. The next day, Friday the 19th, Hilbert congratulates Einstein for having mastered the perihelion problem and adds cheerfully: "If I could calculate as quickly as you, then the electron would have to capitulate in the face of my equations and at the same time the hydrogen atom would have to offer its excuses for the fact that it does not radiate" (Pais 82, p.260).

On 20 November Hilbert presents to the Gesellschaft der Wissenschaften in Göttingen his work. He derives the correct equations from the variational principle assuming general covariance and a second order equation for $g_{\mu\nu}$. He gives full credit to Einstein's ideas. On the first page of his article he writes: "Einstein . . . has brought forth profound thoughts and unique conceptions, and has invented methods for dealing with them . . . Following the axiomatic method, in fact from two simple axioms, I would like to propose a new system of the basic equations of physics. They are of ideal beauty and I believe they solve the problems of Einstein and Mie at the same time". In the published version Hilbert refers to all Einstein November papers. About the one of 25 November, submitted after his talk, he says: "It seems to me that [our] differential equations of gravitation are in agreement with the noble theory of general relativity proposed by Einstein in his later memoire".

On 25 November Einstein proposes **without derivation** the equation

$R_{\mu\nu} = \kappa(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu})$, (Einstein)

which is exactly equivalent to Hilbert's Eq., since they both imply $R + T = 0$. He chooses not to mention Hilbertfs name in the published paper.

Later commentators have a hard time to understand what was Einsteinfs argument at the time to include the trace term. Only Norton makes a well documented (59 pages long) case (including the study of a Zürich notebook of Einstein) for an independent Einsteinfs road to the correct equations.

Einstein papers

serve as a counterargument because the first term on its right-hand side can be brought into the form

$$\sum_{\nu\tau} \left\{ {\sigma\nu \atop \tau} \right\} T_\tau^\nu .$$

Therefore, from now on we shall call the quantities

$$\Gamma_{\mu\nu}^\sigma = -\left\{ {\mu\nu \atop \sigma} \right\} = -\sum_\alpha g^{\sigma\alpha} \left[{\mu\nu \atop \alpha} \right] = -\frac{1}{2} \sum_\alpha g^{\sigma\alpha} \left(\frac{\partial g_{\mu\alpha}}{\partial x_\nu} + \frac{\partial g_{\nu\alpha}}{\partial x_\mu} - \frac{\partial g_{\mu\nu}}{\partial x_\alpha} \right) \tag{15}$$

the components of the gravitational field. K_ν vanishes when T_σ^ν denotes the energy tensor of all "material" processes, and the conservation theorem (14) takes the form

$$\sum_\alpha \frac{\partial T_\sigma^\alpha}{\partial x_\alpha} = -\sum_{\alpha\beta} \Gamma_{\sigma\beta}^\alpha T_\alpha^\beta . \tag{14a}$$

We note that the equations of motion (23b) l.c. of a material point in a gravitational field take the form

$$\frac{d^2 x_\tau}{ds^2} = \sum_{\mu\nu} \Gamma_{\mu\nu}^\tau \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} . \tag{15}$$ {2}

2. The considerations in paragraphs 10 and 11 of the quoted paper remain unchanged, except that the structures which were there called *V*-scalars and *V*-tensors are now ordinary scalars and tensors, respectively.

§3. The Field Equations of Gravitation

From what has been said, it seems appropriate to write the field equations of gravitation in the form

$$R_{\mu\nu} = -\kappa T_{\mu\nu} \tag{16}$$

since we already know that these equations are covariant under any transformation of a determinant equal to 1. Indeed, these equations satisfy all conditions we can demand. Written out in more detail, and according to (13a) and (15), they are

$$\sum_\alpha \frac{\partial \Gamma_{\mu\nu}^\alpha}{\partial x_\alpha} + \sum_{\alpha\beta} \Gamma_{\mu\beta}^\alpha \Gamma_{\nu\alpha}^\beta = -\kappa T_{\mu\nu} . \tag{16a}$$

Based upon this system one can—by retroactive choice of coordinates—return to those laws which I established in my recent paper, and without any actual change in these laws, because it is clear that we can introduce a new coordinate system such that relative to it

$$\sqrt{-g} = 1$$

holds everywhere. S_{im} then vanishes and one returns to the system of field equations

$$R_{\mu\nu} = -\kappa T_{\mu\nu} \tag{16}$$

of the recent paper. The formulas of absolute differential calculus degenerate exactly in the manner shown in said paper. And our choice of coordinates still allows only transformations of determinant 1.

The only difference in content between the field equations derived from general covariance and those of the recent paper is that the value of $\sqrt{-g}$ could not be prescribed in the latter. This value was rather determined by the equation

{1} $$\sum_{\alpha\beta} \frac{\partial}{\partial x_\alpha}\left(g^{\alpha\beta} \frac{\partial lg\sqrt{-g}}{\partial x_\beta}\right) = -\kappa \sum_\sigma T_\sigma^\sigma. \tag{21a}$$

This equation shows that here $\sqrt{-g}$ can only be constant if the scalar of the energy tensor vanishes.

Under our present derivation $\sqrt{-g} = 1$ due to our arbitrary choice of coordinates. The vanishing of the scalar of the energy tensor of "matter" follows now from our field equations instead of from equation (21a). *The generally covariant field equations (16b), which form our starting point, do not lead to a contradiction only when the hypothesis, which we explained in the introduction, applies.* Then, however, we are also entitled to add to our previous field equation the limiting condition:

$$\sqrt{-g} = 1. \tag{21b}$$

Additional note by translator

{1} The "∂x_α" inside of the parentheses has been corrected to "∂x_β."

Doc. 25

[p. 844] Session of the physical-mathematical class on November 25, 1915

The Field Equations of Gravitation

by A. Einstein

In two recently published papers[1] I have shown how to obtain field equations of gravitation that comply with the postulate of general relativity, i.e., which in their general formulation are covariant under arbitrary substitutions of space-time variables.

Historically they evolved in the following sequence. First, I found equations that contain the NEWTONIAN theory as an approximation and are also covariant under arbitrary substitutions of determinant 1. Then I found that these equations are equivalent to generally-covariant ones if the scalar of the energy tensor of "matter" vanishes. The coordinate system could then be specialized by the simple rule that $\sqrt{-g}$ must equal 1, which leads to an immense simplification of the equations of the theory. It has to be mentioned, however, that this requires the introduction of the hypothesis that the scalar of the energy tensor of matter vanishes.

I now quite recently found that one can get away without this hypothesis about the energy tensor of matter merely by inserting it into the field equations in a slightly different way. The field equations for vacuum, onto which I based the explanation of the Mercury perihelion, remain unaffected by this modification. In order not to [2]
force the reader constantly to consult the previous publications, I repeat here the considerations in their entirety.

One derives from the well-known RIEMANN-covariant of rank four the following covariant of rank two:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix}. \tag{1b}$$

[1]*Sitzungsber.* 44, p. 778, and 46, p. 799 (1915). [1]

[p. 845] The ten generally-covariant equations of the gravitational field in spaces where "matter" is absent are obtained by setting

$$G_{im} = 0. \tag{2}$$

These equations can be simplified by choosing the system of reference such that $\sqrt{-g} = 1$. S_{im} then vanishes because of (16), and one gets instead of (2)

$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^l_{i\rho} \Gamma^\rho_{ml} = 0 \tag{3}$$

$$\sqrt{-g} = 1. \tag{3a}$$

We have set here

$$\Gamma^l_{im} = -\left\{ \begin{matrix} im \\ l \end{matrix} \right\}, \tag{4}$$

which quantities we call the "components" of the gravitational field.

When there is "matter" in the space under consideration, its energy tensor occurs on the right-hand sides of (2) and (3), respectively. We set

$$G_{im} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right), \tag{2a}$$

where

$$\sum_{\rho\sigma} g^{\rho\sigma} T_{\rho\sigma} = \sum_\sigma T^\sigma_\sigma = T. \tag{5}$$

T is the scalar of the energy tensor of "matter," and the right-hand side of (2a) is a tensor. If we specialize the coordinate system again in the familiar manner, we get in place of (2a) the equivalent equations

$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^l_{i\rho} \Gamma^\rho_{ml} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right) \tag{6}$$

$$\sqrt{-g} = 1. \tag{3a}$$

We assume, as usual, that the divergence of the energy tensor of matter vanishes when taken in the sense of the general differential calculus (energy-momentum theorem). Specializing the choice of coordinates according to (3a), this means basically that the T_{im} should satisfy the conditions

$$\sum_\lambda \frac{\partial T^\lambda_\sigma}{\partial x_\lambda} = -\frac{1}{2} \sum_{\mu\nu} \frac{\partial g^{\mu\nu}}{\partial x_\sigma} T_{\mu\nu} \tag{7}$$

or

$$\sum_{\lambda} \frac{\partial T_{\sigma}^{\lambda}}{\partial x_{\lambda}} = -\sum_{\mu\nu} \Gamma_{\sigma\nu}^{\mu} T_{\mu}^{\nu}. \tag{7a}$$

When one multiplies (6) by $\partial g^{im}/\partial x_{\sigma}$ and sums over i and m, one gets[2] because [p. 846]
of (7) and because of the relation

$$\frac{1}{2}\sum_{im} g_{im} \frac{\partial g^{im}}{\partial x_{\sigma}} = -\frac{\partial lg\sqrt{-g}}{\partial x_{\sigma}} = 0$$

that follows from (3a), the conservation theorem of matter and gravitational field combined in the form

$$\sum_{\lambda} \frac{\partial}{\partial x_{\lambda}} (T_{\sigma}^{\lambda} + t_{\sigma}^{\lambda}) = 0, \tag{8}$$

where t_{σ}^{λ} (the "energy tensor" of the gravitational field) is given by

$$\kappa t_{\sigma}^{\lambda} = \frac{1}{2}\delta_{\sigma}^{\lambda} \sum_{\mu\nu\alpha\beta} g^{\mu\nu} \Gamma_{\mu\beta}^{\alpha} \Gamma_{\nu\alpha}^{\beta} - \sum_{\mu\nu\alpha} g^{\mu\nu} \Gamma_{\mu\sigma}^{\alpha} \Gamma_{\nu\alpha}^{\lambda}. \tag{8a}$$

The reasons that motivated me to introduce the second term on the right-hand sides of (2a) and (6) will only become transparent in what follows, but they are completely analogous to those just quoted (p. 785).

When we multiply (6) by g^{im} and sum over i and m, we obtain after a simple calculation

$$\sum_{\alpha\beta} \frac{\partial^2 g^{\alpha\beta}}{\partial x_{\alpha} \partial x_{\beta}} - \kappa(T + t) = 0, \tag{9}$$

where, corresponding to (5), we used the abbreviation

$$\sum_{\rho\sigma} g^{\rho\sigma} t_{\rho\sigma} = \sum_{\sigma} t_{\sigma}^{\sigma} = t. \tag{8b}$$

It should be noted that our additional term is such that the energy tensor of the gravitational field occurs in (9) on footing equal with the one of matter, which was not the case in equation (21) l.c.

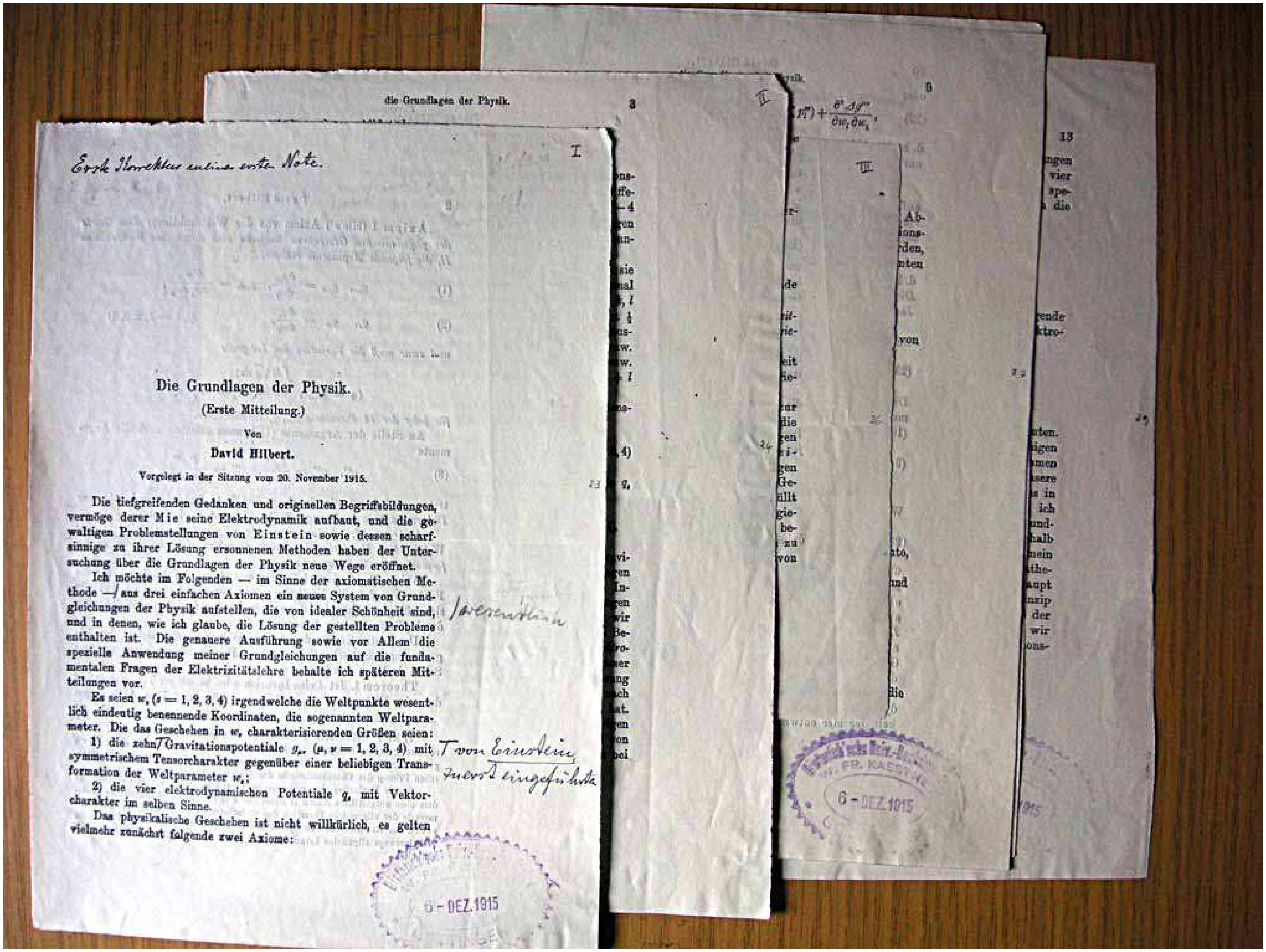
Furthermore, one derives in place of equation (22) l.c. and in the same manner as there, with the help of the energy equation, the relations

[2]On the derivation see *Sitzungsber.* 44 (1915), pp. 784—785. For the following I ask [3]
the reader also to consult, for a comparison, the deliberations given there on p. 785.

$$\frac{\partial}{\partial x_\mu}\left[\sum_{\alpha\beta}\frac{\partial^2 g^{\alpha\beta}}{\partial x_\alpha \partial x_\beta} - k(T + t)\right] = 0. \tag{10}$$

Our additional term insures that these equations carry no additional conditions when compared to (9); we thus need not make other hypotheses about the energy tensor of [p. 847] matter other than that it complies with the energy momentum theorem.

With this, we have finally completed the general theory of relativity as a logical structure. The postulate of relativity in its most general formulation (which makes space-time coordinates into physically meaningless parameters) leads with compelling necessity to a very specific theory of gravitation that also explains the movement of the perihelion of Mercury. However, the postulate of general relativity cannot reveal to us anything new and different about the essence of the various processes in nature than what the special theory of relativity taught us already. The opinions I recently [4] voiced here in this regard have been in error. Every physical theory that complies with the special theory of relativity can, by means of the absolute differential calculus, be integrated into the system of general relativity theory—without the latter providing any criteria about the admissibility of such physical theory.



Die Grundlagen der Physik.

(Erste Mitteilung.)

Von

David Hilbert.

Vorgelegt in der Sitzung vom 20. November 1915.

Die tiefgreifenden Gedanken und originellen Begriffsbildungen, vermöge derer Mie seine Elektrodynamik aufbaut, und die gewaltigen Problemstellungen von Einstein sowie dessen scharfsinnige zu ihrer Lösung ersonnenen Methoden haben der Untersuchung über die Grundlagen der Physik neue Wege eröffnet.

Ich möchte im Folgenden — im Sinne der axiomatischen Methode — aus drei einfachen Axiomen ein neues System von Grundgleichungen der Physik aufstellen, die von idealer Schönheit sind, und in denen, wie ich glaube, die Lösung der gestellten Probleme enthalten ist. Die genauere Ausführung sowie vor Allem die spezielle Anwendung meiner Grundgleichungen auf die fundamentalen Fragen der Elektrizitätslehre behalte ich späteren Mitteilungen vor.

Es seien w_s $(s = 1, 2, 3, 4)$ irgendwelche die Weltpunkte wesentlich eindeutig benennende Koordinaten, die sogenannten Weltparameter. Die das Geschehen in w_s charakterisierenden Größen seien:

1) die zehn Gravitationspotentiale $g_{\mu\nu}$ $(\mu, \nu = 1, 2, 3, 4)$ mit symmetrischem Tensorcharakter gegenüber einer beliebigen Transformation der Weltparameter w_s;

2) die vier elektrodynamischen Potentiale q_s mit Vektorcharakter im selben Sinne.

Das physikalische Geschehen ist nicht willkürlich, es gelten vielmehr zunächst folgende zwei Axiome:

Da K nur von $g^{\mu\nu}$, $g^{\mu\nu}_s$, $g^{\mu\nu}_{lk}$ abhängt, so läßt sich beim Ansatz (17) die Energie E wegen (13) lediglich als Funktion der Gravitationspotentiale $g^{\mu\nu}$ und deren Ableitungen ausdrücken, sobald wir L nicht von $g^{\mu\nu}_s$, sondern nur von $g^{\mu\nu}$, q_s, q_{sk} abhängig annehmen. Unter dieser Ausnahme, die wir im Folgenden stets machen, liefert die Definition der Energie (10) den Ausdruck

$$E = E^g + E^e \tag{18}$$

wo die „Gravitationsenergie" E^g nur von $g^{\mu\nu}$ und deren Ableitungen abhängt und die „elektrodynamische Energie" E^e die Gestalt erhält

$$E^e = \sum_{\mu,\nu,s} \frac{\partial \sqrt{g}\, L}{\partial g^{\mu\nu}} (g^{\mu\nu}_s p^s - g^{\mu s} p^\nu_s - g^{\nu s} p^\mu_s), \tag{19}$$

in der sie sich als *eine mit $\sqrt{g}$ multiplizierte allgemeine Invariante* erweist.

Des Weiteren benutzen wir zwei mathematische Theoreme, die wie folgt lauten:

Theorem II. Wenn J eine von $g^{\mu\nu}$, $g^{\mu\nu}_l$, $g^{\mu\nu}_{lk}$, q_s, q_{sk} abhängige Invariante ist, so gilt stets identisch in allen Argumenten und für jeden willkürlichen kontravarianten Vektor p^s

$$\sum_{\mu,\nu,l,k} \left(\frac{\partial J}{\partial g^{\mu\nu}} \Delta g^{\mu\nu} + \frac{\partial J}{\partial g^{\mu\nu}_l} \Delta g^{\mu\nu}_l + \frac{\partial J}{\partial g^{\mu\nu}_{lk}} \Delta g^{\mu\nu}_{lk} \right) + \sum_{s,k} \left(\frac{\partial J}{\partial q_s} \Delta q_s + \frac{\partial J}{\partial q_{sk}} \Delta q_{sk} \right) = 0;$$

dabei ist

$$\Delta g^{\mu\nu} = \sum_m (g^{\mu m} p^\nu_m + g^{\nu m} p^\mu_m),$$

$$\Delta g^{\mu\nu}_l = -\sum_m g^{\mu\nu}_m p^m_l + \frac{\partial \Delta g^{\mu\nu}}{\partial w_l},$$

Erste Korrektur meiner ersten Note. I

Die Grundlagen der Physik.

(Erste Mitteilung.)

Von

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Vorgelegt in der Sitzung vom 20. November 1915. 23

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1) die zehn ⊤ Gravitationspotentiale $g_{\mu\nu}$ $(\mu, \nu = 1, 2, 3, 4)$ mit symmetrischem Tensorcharakter gegenüber einer beliebigen Transformation der Weltparameter w_s; *⊤ von Einstein zuerst eingeführte*

2) die vier elektrodynamischen Potentiale q_s mit Vektorcharakter im selben Sinne.

Das physikalische Geschehen ist nicht willkürlich, es gelten vielmehr zunächst folgende zwei Axiome:

Axiom I (Mie's[1]) Axiom von der Weltfunktion): *Das Gesetz des physikalischen Geschehens bestimmt sich durch eine Weltfunktion H, die folgende Argumente enthält:*

$$(1)\qquad g_{\mu\nu},\quad g_{\mu\nu l}=\frac{\partial g_{\mu\nu}}{\partial w_l},\quad g_{\mu\nu lk}=\frac{\partial^2 g_{\mu\nu}}{\partial w_l\,\partial w_k},$$

$$(2)\qquad q_s,\quad q_{sl}=\frac{\partial q_s}{\partial w_l},\qquad (l,k=1,2,3,4)$$

und zwar muß die Variation des Integrals

$$\int H\sqrt{g}\,d\tau;$$

$$(g=|g_{\mu\nu}|,\quad d\tau=dw_1\,dw_2\,dw_3\,dw_4)$$

für jedes der 14 Potentiale $g_{\mu\nu}$, q_s verschwinden.

An Stelle der Argumente (1) können offenbar auch die Argumente

$$(3)\qquad g^{\mu\nu},\quad g^{\mu\nu}_l=\frac{\partial g^{\mu\nu}}{\partial w_l},\quad g^{\mu\nu}_{lk}=\frac{\partial^2 g^{\mu\nu}}{\partial w_l\,\partial w_k}$$

treten, wobei $g^{\mu\nu}$ die durch g dividierte Unterdeterminante der Determinante g in Bezug auf ihr Element $g_{\mu\nu}$ bedeutet.

Axiom II[2]) (Axiom von der allgemeinen Invarianz): *Die Weltfunktion H ist eine Invariante gegenüber einer beliebigen Transformation der Weltparameter w_s.*

Axiom II ist der einfachste mathematische Ausdruck für die Forderung, daß die Verkettung der Potentiale $g_{\mu\nu}$, q_s an und für sich völlig unabhängig ist von der Art, wie man die Weltpunkte durch Weltparameter benennen will.

Das Leitmotiv für den Aufbau meiner Theorie liefert der folgende mathematische Satz, dessen Beweis ich an einer anderen Stelle darlegen werde.

— der

Theorem I. Ist J eine Invariante bei beliebiger Transformation der vier Weltparameter, welche n Größen und ihre Ableitungen

1) Mie's Weltfunktionen enthalten nicht genau diese Argumente; insbesondere geht der Gebrauch der Argumente (2) auf Born zurück; es ist jedoch gerade die Einführung und Verwendung einer solchen Weltfunktion im Hamiltonschen Prinzip das Charakteristische der Mie'schen Elektrodynamik.

2) Die Forderung der orthogonalen Invarianz hat bereits Mie gestellt. In dem oben aufgestellten Axiom II findet der Einsteinsche Grundgedanke fundamentale der allgemeinen Invarianz den einfachsten Ausdruck, wennschon bei Einstein das Hamiltonsche Prinzip nur eine Nebenrolle spielt und seine Funktionen H keineswegs allgemeine Invarianten sind, auch die elektrischen Potentiale nicht enthalten.

III

Dieser Satz zeigt, daß die dem Energiesatz der alten Theorie entsprechende Divergenzgleichung

$$(15)\qquad \sum_l \frac{\partial e^l_s}{\partial w_l}=0$$

dann und nur dann gelten kann, wenn die vier Größen e_s verschwinden, d. h. wenn die Gleichungen gelten

$$(16)\qquad \frac{\partial^{\,g}\sqrt{g}\,H}{\partial w_s}=0$$

Nach diesen Vorbereitungen stelle ich nunmehr das folgende Axiom auf:

Axiom III (Axiom von Raum und Zeit). *Die Raum-Zeitkoordinaten sind solche besonderen Weltparameter, für die der Energiesatz (15) gültig ist.*

Nach diesem Axiom liefern in Wirklichkeit Raum und Zeit eine solche besondere Benennung der Weltpunkte, daß der Energiesatz gültig ist.

Das Axiom III hat das Bestehen der Gleichungen (16) zur Folge: diese vier Differentialgleichungen (16) vervollständigen die Gravitationsgleichungen (4) zu einem System von 14 Gleichungen für die 14 Potentiale $g^{\mu\nu}$, q_s: *dem System der Grundgleichungen der Physik.* Wegen der Gleichzahl der Gleichungen und der zu bestimmenden Potentiale ist für das physikalische Geschehen auch das Kausalitätsprinzip gewährleistet, und es enthüllt sich uns damit der engste Zusammenhang zwischen dem Energiesatz und dem Kausalitätsprinzip, indem beide sich einander bedingen. Dem Übergang von einem Raum-Zeit-Bezugssystem zu einem anderen entspricht die Transformation der Energieform von einer sogenannten „Normalform"

$$E=\sum_{s,l} e^l_s E^s_l$$

auf eine andere Normalform.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Axiom I (Mie's[1]) Axiom von der Weltfunktion): *Das Gesetz des physikalischen Geschehens bestimmt sich durch eine Weltfunktion H, die folgende Argumente enthält:*

$$g_{\mu\nu},\quad g_{\mu\nu l} = \frac{\partial g_{\mu\nu}}{\partial w_l},\quad g_{\mu\nu lk} = \frac{\partial^2 g_{\mu\nu}}{\partial w_l \partial w_k}, \tag{1}$$

$$q_s,\quad q_{sl} = \frac{\partial q_s}{\partial w_l}, \qquad (l, k = 1, 2, 3, 4) \tag{2}$$

und zwar muß die Variation des Integrals

Da K nur von $g^{\mu\nu}$, $g^{\mu\nu}_s$, $g^{\mu\nu}_{lk}$ abhängt, so läßt sich beim Ansatz (17) die Energie E wegen (13) lediglich als Funktion der Gravitationspotentiale $g^{\mu\nu}$ und deren Ableitungen ausdrücken, sobald wir L nicht von $g^{\mu\nu}_s$, sondern nur von $g^{\mu\nu}$, q_s, q_{sk} abhängig annehmen. Unter dieser Ausnahme, die wir im Folgenden stets machen, liefert die Definition der Energie (10) den Ausdruck

$$E = E^{(g)} + E^{(e)} \tag{18}$$

wo die „Gravitationsenergie" $E^{(g)}$ nur von $g^{\mu\nu}$ und deren Ableitungen abhängt und die „elektrodynamische Energie" $E^{(e)}$ die Gestalt erhält

$$E^{(e)} = \sum_{\mu,\nu,s} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} (g^{\mu\nu}_s p^s - g^{\mu s} p^\nu_s - g^{\nu s} p^\mu_s), \tag{19}$$

in der sie sich als *eine mit* $\sqrt{g}$ *multiplizierte allgemeine Invariante* erweist.

Des Weiteren benutzen wir zwei mathematische Theoreme, die wie folgt lauten:

Theorem II. Wenn J eine von $g^{\mu\nu}$, $g^{\mu\nu}_l$, $g^{\mu\nu}_{lk}$, q_s, q_{sk} abhängige Invariante ist, so gilt stets identisch in allen Argumenten und für jeden willkürlichen kontravarianten Vektor p^s

$$\sum_{\mu,\nu,l,k} \left(\frac{\partial J}{\partial g^{\mu\nu}} \Delta g^{\mu\nu} + \frac{\partial J}{\partial g^{\mu\nu}_l} \Delta g^{\mu\nu}_l + \frac{\partial J}{\partial g^{\mu\nu}_{lk}} \Delta g^{\mu\nu}_{lk} \right) + \sum_{s,k} \left(\frac{\partial J}{\partial q_s} \Delta q_s + \frac{\partial J}{\partial q_{sk}} \Delta q_{sk} \right) = 0;$$

dabei ist

$$\Delta g^{\mu\nu} = \sum_m (g^{\mu m} p^\nu_m + g^{\nu m} p^\mu_m),$$

$$\Delta g^{\mu\nu}_l = -\sum_m g^{\mu\nu}_m p^m_l + \frac{\partial \Delta g^{\mu\nu}}{\partial w_l},$$

Dieser Satz zeigt, daß die dem Energiesatz der alten Theorie entsprechende Divergenzgleichung

$$(15) \qquad \sum_l \frac{\partial e_s^l}{\partial w_l} = 0$$

dann und nur dann gelten kann, wenn die vier Größen r_s verschwinden, d. h. wenn die Gleichungen gelten

$$(16) \qquad \frac{d^{w}\sqrt{g}\,H}{dw_s} = 0$$

Nach diesen Vorbereitungen stelle ich nunmehr das folgende Axiom auf:

Axiom III (Axiom von Raum und Zeit). *Die Raum-Zeitkoordinaten sind solche besonderen Weltparameter, für die der Energiesatz* (15) *gültig ist.*

Nach diesem Axiom liefern in Wirklichkeit Raum und Zeit eine solche besondere Benennung der Weltpunkte, daß der Energiesatz gültig ist.

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$$E = \sum_{s,l} e_s^l E_l^s$$

auf eine andere Normalform.

Da K nur von $g^{\mu\nu}, g_s^{\mu\nu}, g_{lk}^{\mu\nu}$ abhängt, so läßt sich beim Ansatz (17) die Energie E wegen (13) lediglich als Funktion der Gravitationspotentiale $g^{\mu\nu}$ und deren Ableitungen ausdrücken, sobald wir L nicht von $g_s^{\mu\nu}$, sondern nur von $g^{\mu\nu}, q_s, q_{sk}$ abhängig annehmen. Unter dieser Annahme, die wir im Folgenden stets machen, liefert die Definition der Energie (10) den Ausdruck

$$E = E^g + E^e \tag{18}$$

wo die „Gravitationsenergie" E^g nur von $g^{\mu\nu}$ und deren Ableitungen abhängt und die „elektrodynamische Energie" E^e die Gestalt erhält

$$E^e = \sum_{\mu,\nu,s} \frac{\partial\sqrt{g}\,L}{\partial g^{\mu\nu}}\left(g_s^{\mu\nu} p^s - g^{\mu s} p_s^\nu - g^{\nu s} p_s^\mu\right), \tag{19}$$

in der sie sich als *eine mit $\sqrt{g}$ multiplizierte allgemeine Invariante* erweist.

Des Weiteren benutzen wir zwei mathematische Theoreme, die wie folgt lauten:

Theorem II. Wenn J eine von $g^{\mu\nu}, g_l^{\mu\nu}, g_{lk}^{\mu\nu}, q_s, q_{sk}$ abhängige Invariante ist, so gilt stets identisch in allen Argumenten und für jeden willkürlichen kontravarianten Vektor p^s

$$\sum_{\mu,\nu,l,k}\left(\frac{\partial J}{\partial g^{\mu\nu}}\Delta g^{\mu\nu} + \frac{\partial J}{\partial g_l^{\mu\nu}}\Delta g_l^{\mu\nu} + \frac{\partial J}{\partial g_{lk}^{\mu\nu}}\Delta g_{lk}^{\mu\nu}\right)$$
$$+\sum_{s,k}\left(\frac{\partial J}{\partial q_s}\Delta q_s + \frac{\partial J}{\partial q_{sk}}\Delta q_{sk}\right) = 0;$$

dabei ist

$$\Delta g^{\mu\nu} = \sum_m \left(g^{\mu m} p_m^\nu + g^{\nu m} p_m^\mu\right),$$
$$\Delta g_l^{\mu\nu} = -\sum_m g_m^{\mu\nu} p_l^m + \frac{\partial \Delta g^{\mu\nu}}{\partial w_l},$$

Nov 16, 1915 Hilbert writes Einstein a postcard, which is now lost. So we do not know the exact date, which could vary by one day, nor exactly what was written on it. According to Wuensch[21] the postcard contained the following three formulae

$$H = K + L \tag{5a}$$

$$\delta \int (K + L)\sqrt{g}\, d\omega = 0 \tag{5b}$$

$$\sqrt{g}\,(K_{\mu\nu} - \tfrac{1}{2}K\,g_{\mu\nu}) + \frac{\partial L\sqrt{g}}{\partial g^{\mu\nu}} = 0 \tag{5c}$$

(5c) are the correct field equations of General Relativity, including the trace term $-\frac{1}{2}K\,g_{\mu\nu}$. The

1997 – till now

Summing up the decisive phase of his work on general relativity (A. Fölsing, Albert Einstein: A Biography (Viking, N.Y. 1997)) quotes Einstein's letter to Heinrich Zangger (see also an earlier discussion of this letter in (Med 84)) which says: "Only one colleague truly understood it, and he now tries skillfully to "nostrify it" [i.e. appropriate ("make it ours")]. We already know that the colleague in question was none other than David Hilbert. Fölsing justly refutes the accusation on the basis of available evidence.

Later the same year an article in the 14 November issue of Science, made the news. This paper has a direct bearing on our topic. It points out that a lately discovered proof-sheet of Hilbert's paper, with a publisher's stamp of 6 December 1915, i.e. after the publication of the fourth of Einstein's communications, involves substantial changes in the manuscript. The fact that Hilbert modified his paper after its submission has been known before: as we noted he had cited all four Einstein's November papers and had commented on the last one (submitted after his) in the published version of his November 20 article. The authors strive to attribute a great significance to the fact that the original text only involves the Hilbert action, while the field equations, which are derived from it, appear to be first inserted at the stage of the proofreading. Their attempt to support on this ground Einstein's accusation of "nostrification" goes much too far. A calm, non-confrontational reaction was soon provided by a thorough study of Hilbert's route to the "Foundations of Physics".

The polemics is getting rough. A new book, (Wuensch 05), is advertised with a question mark: "Ein Kriminalfall in der Wissenschaftsgeschichte?" (Wuensch, A criminal case in the history of science?, 2005). The author asserts - already in the abstract to the book - that a missing fragment of the text on pages 7 and 8 of Hilbert's proof-sheets, used in Science, contained "in all probability ... the explicit form of the field equations..." She further argues that "the passage ... was not excised originally but rather ... it must have been deliberately removed in more recent times in order to falsify the historical truth."

I. Todorov: "Einstein and Hilbert had the moral strength and wisdom - after a month of intense competition, from which, in a final account, everybody (including science itself) profited - to avoid a lifelong priority dispute (something in which Leibnitz and Newton failed). It would be a shame to subsequent generations of scientists and historians of science to try to undo their achievement".

Conclusion

- Read critically all available papers and books
- Do NOT trust even famous authors in their references
- Pay special attention to classical papers

Thank you for your attention