Mathematical Visualization Tool GCLC/WinGCLC

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Agenda

- Brief Survey of Dynamic Geometry Software
- Tool GCLC/WinGCLC
- Automated Reasoning in Geometry and GCLC
- Demo

What is Dynamic Geometry Software?

- Interactive geometry software or Dynamic geometry software or Dynamic geometry environments or Dynamic geometry tools
- DG tools allow the user "to create and then manipulate geometric constructions, primarily in plane geometry"
- The user typically starts a construction with a few points, construct new objects, and then can move the points to see how the construction changes

What Good is Dynamic Geometry Software?

- Fun and good for exploring geometry and mathematics
- Good:
  - for students — to explore and understand mathematical objects and notions;
  - teachers — to demonstrate and illustrate concepts;
  - for publishing — for easy production of complex mathematical figures.

Some Dynamic Geometry Tools

- Commercial: Cabri Geometry (since 1988), Geometer Sketchpad (GSP) (since 1991), Cinderella
- Free: KSEG, Eukleides, DrGeo
- 3D: Cabri 3D, Archimedes Geo3D, JavaView
- More details: http://en.wikipedia.org/wiki/Dynamic_geometry_software

Different Tools — Different Skills

- Animations, loci, ...
- Symbolic expressions, calculations, ...
- Saving constructions, saving figures, ...
- Multilingual
- Automated theorem proving, probabilistic proofs, ...

What is GCLC/WinGCLC?

- Developed since 1996, originally, as a tool for producing geometrical illustrations in LaTeX, hence the name GCLC: "Geometry Constructions → LaTeX Converter"
- Today — a general mathematical visualization tool
- Available from: http://www.matf.bg.ac.rs/~janicic/gclc and from EMIS (The European Mathematical Information Service) servers http://www.emis.de/misc/index.html

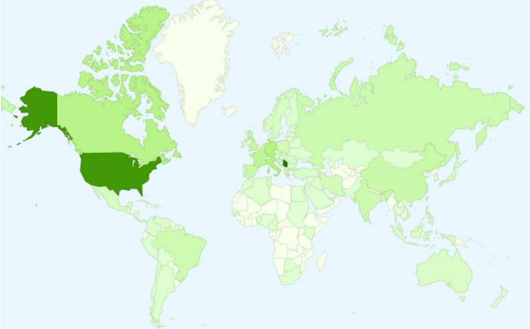
Main applications of GCLC/WinGCLC

Main applications of GCLC/WinGCLC:

- producing digital mathematical illustrations
- mathematical education
- storing mathematical contents
- studies of automated geometrical reasoning

GCLC Users

- Used in a number of high-schools and university courses, and for publishing
- Thousands of users worldwide:



Basic Principles of GCLC

- A construction is a formal procedure, not an image
- Producing mathematical illustrations is based on "describing figures", not on "drawing figures" (similarly as TEX)
- Not WYSIWYG and is not based on point-and-click approach
- All instructions are given explicitly, in GCLC language

GCLC Features (part I)

- Support for geometrical primitive constructions, compound constructions, transformations, etc.
- Symbolic expressions, while-loops, user-defined procedures
- Conics, 2D and 3D curves, 3D surfaces
- Log files with information on all objects
- Built-in theorem provers

GCLC Features (part II)

- WinGCLC: graphical, user-friendly interface, interactive work, animations, traces
- Export to different formats (LaTeX— several versions, EPS, BMP, SVG), import from JavaView
- Full XML support
- Free, small in size (<1Mb), easy to use, well documented

Basics of GCLC Language

- GCLC language is like a simple programming language, easily understandable to mathematicians
- All instructions are explicit, given by GCLC commands
- Instructions for describing **contents**
- Instructions for describing **presentation**

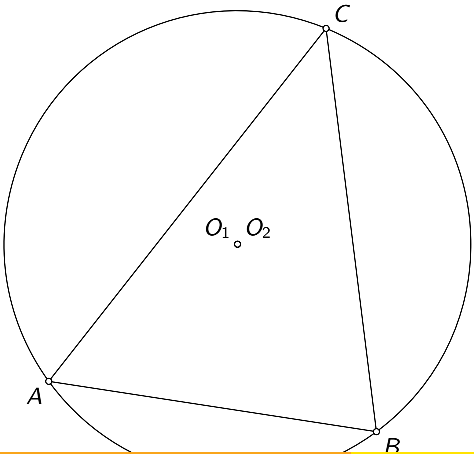
Overview of the GCLC Language

- Basic definitions, constructions, transformations
- Drawing, labelling, and printing commands
- 2D and 3D Cartesian commands
- Symbolic expressions, loops, user-defined procedures
- Commands for describing animations
- Commands for the geometry theorem proving

Simple Example (part I)

```
point A 15 20
point B 80 10
point C 70 90
med a B C
med b A C
med c B A
intersection O_1 a b
intersection O_2 a c
cmark_lb A
cmark_rb B
cmark_rt C
cmark_lt O_1
cmark_rt O_2
drawsegment A B
drawsegment A C
drawsegment B C
drawcircle O_1 A
```

Simple Example (part II)



Using GCLC Images with LaTeX

```
\documentclass{article}
\usepackage{gclc}
...
\begin{document}
...
\input{figure.pic}
...
\end{document}
```

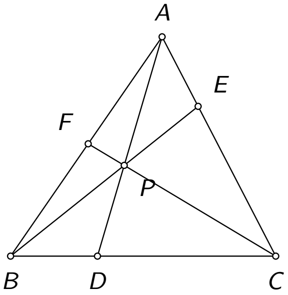
Brief History of Automated Reasoning in Geometry

- Around for more than 50 years
- Early AI-based approaches in 50's
- Algebraic theorem provers — Gröbner-bases Method (Buchberger 1965), Wu's Method (Wu 1977).
- Coordinate-free methods – area method, full-angle, vector method (Chou et.al.1990's)
- Coherent logic based methods (2000's)

Theorem Provers Built-into GCLC

- Gröbner-bases Method
- Wu's Method
- Area method
- All of them are very efficient and can prove hundreds of non-trivial theorems in only milliseconds
- Simple usage: only add e.g., `prove { identical A B }`

Example: Ceva's Theorem



- Conjecture:

$$\frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} = 1$$

- Outputs by the three provers...

Short Demo of WinGCLC